

# GCSE

## Mathematics A

Draft Scheme of Work  
Emporium version

Edexcel GCSE in Mathematics A (1MA0)

For first teaching from September 2010





# Contents

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<b>Introduction</b>	<b>1</b>
<b>Foundation Scheme of Work</b>	<b>3</b>
<b>Foundation Overview</b>	<b>5</b>
<b>Foundation Modules</b>	<b>7</b>
<b>Foundation Objectives</b>	<b>45</b>
<b>Higher Scheme of Work</b>	<b>49</b>
<b>Higher Overview</b>	<b>51</b>
<b>Higher Modules</b>	<b>53</b>
<b>Higher Objectives</b>	<b>91</b>



# Introduction

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This scheme of work is based upon a five term model over two years for both Foundation and Higher tier students. It can be used directly as a scheme of work for the GCSE Mathematics A specification (1MA0).

The scheme of work is structured so each topic contains:

- Module number
- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Tier
- Contents, referenced back to the specification
- Objectives for students at the end of the module
- References to published textbook sections
- Ideas for differentiation and extension activities
- Notes for general mathematical teaching points and common misconceptions

Updates will be available via a link from the Edexcel mathematics website ([www.edexcel.com](http://www.edexcel.com)).

References to Edexcel published student books for the course are given in brackets for each main teaching objective. For example (2.6) in a Foundation module references to GCSE Mathematics A Foundation Student Book, Chapter 2, Section 2.6.



**GCSE Mathematics A (1MA0)  
Foundation  
Tier**

**Linear  
Scheme of Work**



# Foundation course overview

The table below shows an overview of modules in the Linear Foundation tier scheme of work.

Teachers should be aware that the estimated teaching hours are **approximate** and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Integers	7
2	Decimals	4
3	Coordinates	4
4	Angles, lines and triangles	6
5	Reading scales and converting units	5
6	Collecting data	4
7	Charts and graphs	5
8	Symmetry, Similarity and Congruence	4
9	Types of Number	8
10	Introduction to algebra	4
11	Constructions	5
12	Patterns and Sequences	5
13	Properties of quadrilaterals and parallel lines	5
14	Fractions	7
15	Pie charts	3
16	Fractions, Decimals and Percentages	4
17	Applications of Percentages	5
18	Algebra using powers and brackets	4
19	Ratio and Proportion	6
20	Linear equations and inequalities	6
21	Perimeter and Area	7
22	3-D shapes	4
23	Real-life graphs	5
24	Straight line graphs	4
25	Compound Measures	5
26	Timetables and Distance-time graphs	5
27	Volume	5
28	Probability	9
29	Formulae	7
30	Angles properties of Polygons	5
31	Transformations	6
32	Scatter graphs and Correlation	5
33	Averages and range	7
34	Quadratic Graphs	3
35	Trial and Improvement	3
36	Circles	5
37	Pythagoras' Theorem	5
		190 HOURS



**Module 1**

**Time: 6 – 8 hours**

**GCSE Tier: Foundation**

**Contents: Integers**

- N b Order integers
- N u Approximate to specified or appropriate degrees of accuracy
- N a Add, subtract, multiply and divide any integer
- N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
- N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

- The ability to order numbers
- An appreciation of place value
- Experience of the four operations using whole numbers
- Knowledge of integer complements to 10 and to 100
- Knowledge of strategies for multiplying and dividing whole numbers by 2, 4, 5 and 10

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand and order integers **(1.3)**
- Write numbers in words and writing numbers from words **(1.2)**
- Add and subtract integers **(1.4)**
- Recall all multiplication facts to  $10 \times 10$ , and use them to derive quickly the corresponding division facts **(1 intro)**
- Multiply or divide any number by powers of 10 **(1.5)**
- Multiply and divide integers **(1.5)**
- Use inverse operations (use one calculation to find the answer to another) **(10.2)**
- Use brackets and the hierarchy of operations (BIDMAS) **(9.4)**
- Understand ‘reciprocal’ as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is undefined) **(10.2)**
- Add, subtract, multiply and divide negative numbers **(1.7 – 1.9)**
- Round whole numbers to the nearest: 10, 100, 1000, ... **(1.6)**
- Check calculations by rounding, eg  $29 \times 31 \approx 30 \times 30$  **(5.10)**
- Check answers by reverse calculation, eg if  $9 \times 23 = 207$  then  $207 \div 9 = 23$

**DIFFERENTIATION & EXTENSION**

- Estimate answers to calculations involving the four rules of operation
- Directed number work with multi-step calculations
- Encourage effective use of a calculator
- Try investigations with digits 3, 7, 5 and 2 and challenge students to find the biggest number, smallest odd number, the largest sum or product etc

**NOTES**

- Present all working clearly
- For non-calculator methods, ensure that remainders are shown as evidence of working
- Show what is entered into your calculator, not just the answer
- Try different methods from traditional ones eg Russian or Chinese methods for multiplication
- Incorporate functional elements whenever and wherever possible and always round measures to an appropriate degree of accuracy

**Module** 2

**Time: 3 – 5 hours**

**GCSE Tier: Foundation**

**Contents: Decimals**

- N a Add, subtract, multiply and divide any number  
N j Use decimal notation and recognise that each terminating decimal is a fraction  
N u Approximate to specified or appropriate degrees of accuracy

**PRIOR KNOWLEDGE:**

The concepts of a decimal  
The four operations

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand place value, identifying the values of the digits (5.1)
- Write decimals in ascending order of size (5.2)
- Approximate decimals to a given number of decimal places or to one significant figure (5.7 – 5.9)
- Add and subtract decimals (5.3)
- Multiply and divide decimal numbers by integers and decimal numbers (5.4 – 5.6)
- Know that, eg  $13.5 \div 0.5 = 135 \div 5$
- Check their answers by rounding, and know that eg  $29 \times 31 \approx 30 \times 30$   $31 \approx 30 \times 30$

**DIFFERENTIATION & EXTENSION**

Practise long multiplication and division without using a calculator  
Mental maths problems with negative powers of 10 eg  $2.5 \times 0.01$ ,  $0.001$   
Directed number work with decimal numbers  
Use decimals in real-life problems as much as possible eg Best Buys  
Use functional examples such as entry into theme parks, cost of holidays, sharing the cost of a meal  
Money calculations that require rounding answers to the nearest penny  
Multiply and divide decimals by decimals with more than 2 d.p.  
Round answers to appropriate degrees of accuracy to suit the context of the question

**NOTES**

Advise students not to round decimals, used in calculations, until stating in the final answer  
For non-calculator methods ensure that remainders are shown as evidence of working  
Students need to be clear about the difference between decimal places and significant figures  
Link decimals to Statistics and Probability, eg the mean should not be rounded, the probability of all events occurring is equal to 1  
Link decimals to *Reading scales and converting units (module 5)* and *Compound measures (module 25)*

**Module** 3

**Time:** 3 – 5 hours

**GCSE Tier:** Foundation

**Contents:** Coordinates

A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

**PRIOR KNOWLEDGE:**

Directed numbers  
Parallel and perpendicular lines

**OBJECTIVES**

By the end of the module the student should be able to:

- Use axes and coordinates to specify points in all four quadrants in 2-D (15.1, 15.2)
- Identify points with given coordinates (15.1, 15.2)
- Identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants) (15.1, 15.2)
- Find the coordinates of points identified by geometrical information in 2-D (15.1, 15.2)
- Find the coordinates of the midpoint of a line segment,  $AB$ , given the coordinates of  $A$  and  $B$  (15.3)

**DIFFERENTIATION & EXTENSION**

There are plenty of sources of good material here such as animal pictures with coordinates, games like Connect 4 using coordinates

This topic can be delivered in conjunction with the properties of quadrilaterals

**NOTES**

Clear presentation of graphs with axes correctly labelled is important

GCSE Tier: Foundation

Contents: Angles, lines and triangles

- GM a Recall and use properties of angles at a point, angles at a point on a straight line (including right angles), perpendicular lines, and vertically opposite angles
- GM b Understand and use the angle properties of triangles
- GM t Measure and draw lines and angles
- GM u Draw triangles and other 2-D shapes using a ruler and a protractor

**PRIOR KNOWLEDGE:**

An understanding of angles as a measure of turning  
The ability to use a ruler and a protractor

**OBJECTIVES**

By the end of the module the student should be able to:

- Measure and draw lines, to the nearest mm (2.5)
- Measure and draw angles, to the nearest degree (2.1 – 2.3, 2.5, 2.6)
- Estimate sizes of angles (2.4)
- Recall and use properties of angles: (2.8)
  - at a point
  - angles at a point on a straight line, including right angles
  - vertically opposite angles
- Find the size of missing angles at a point or at a point on a straight line (2.8)
- Distinguish between acute, obtuse, reflex and right angles (2.1 – 2.3)
- Name angles (2.1 – 2.3)
- Give reasons for calculations (2.8)
- Use geometric language appropriately
- Identify points, lines and angles (2.3)
- Use two letter notation for a line and three letter notation for an angle (2.3)
- Recall and use properties of perpendicular lines (7.5)
- Mark perpendicular lines on a diagram (7.5)
- Understand the proof that the angle sum of a triangle is  $180^\circ$  (6.1)
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices (7.3)
- Distinguish between scalene, equilateral, isosceles and right-angled triangles (2.7, 6.1)
- Understand and use the angle properties of triangles (6.1)
- Find a missing angle in a triangle, using the angle sum of a triangle is  $180^\circ$  (6.1)
- Use the side/angle properties of isosceles and equilateral triangles (6.1)
- Understand and use the angle sum of triangles (6.1)
- Make accurate drawing of triangles and other 2-D shapes using a ruler and a protractor (18.2)
- Make an accurate scale drawing from a diagram (7.9)

**DIFFERENTIATION & EXTENSION**

Explore other angle properties in triangles, parallel lines or quadrilaterals, in preparation for future topics

**NOTES**

Make sure that drawings are neat, accurate and labelled  
Give students a lot of drawing practice, and encourage students to check their drawings  
Angles should be accurate to within  $2^\circ$  and lengths accurate to the nearest mm

Module 5

Time: 4 – 6 hours

GCSE Tier: Foundation

Contents: Reading scales and converting units

GM o Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements  
GM t Measure and draw lines  
GM p Convert measurements from one unit to another  
GM m Use scale drawings

**PRIOR KNOWLEDGE:**

An awareness of the imperial system of measures  
Strategies for multiplying and dividing by 10 (for converting metric units)

**OBJECTIVES**

By the end of the module the student should be able to:

- Interpret scales on a range of measuring instruments inc mm, cm, m, km, ml, cl, l, mg, g, kg, tonnes, °C (11.1)
- Indicate given values on a scale (11.1)
- Know that measurements using real numbers depend upon the choice of unit (11.1)
- Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction (11.3, 11.4)
- Convert units within one system (11.3)
- Convert metric units to metric units (Metric equivalents should be known) (11.4)
- Convert imperial units to imperial units (NB: Conversion between imperial units will be given) (11.4)
- Convert between metric and imperial measures (11.4)
- Know rough metric equivalents of pounds, feet, miles, pints and gallons, ie (11.4)
  - Metric Imperial**
  - 1 kg = 2.2 pounds
  - 1.75 pints = 1 litre
  - 4.5 l = 1 gallon
  - 8 km = 5 miles
  - 30 cm = 1 foot
- Estimate conversions (11.4)

**DIFFERENTIATION & EXTENSION**

This could be made a practical activity, by collecting assorted everyday items and weighing and measuring to check the estimates of their lengths, weights and volumes  
Use the internet to find the weights, volumes and heights of large structures such as buildings, aeroplanes and ships  
Take the opportunity to do some real measuring/estimating around school  
Use conversions for height and weight of students, cars, bridges. Combine with simple scales such as 1 cm to 1 m for classrooms, playing fields, bedrooms and ask them to draw a plan of their ideal design for their bedrooms including the furniture

**NOTES**

Measurement is essentially a practical activity  
Use a range of everyday objects to bring reality to lessons  
Use functional elements as a source of practical activities

GCSE Tier: Foundation

Contents: Collecting data

- SP a Understand and use statistical problem solving process/handling data cycle  
 SP b Identify possible sources of bias  
 SP c Design an experiment or survey  
 SP d Design data-collection sheets distinguishing between different types of data  
 SP e Extract data from printed tables and lists  
 SP f Design and use two-way tables for discrete and grouped data

**PRIOR KNOWLEDGE:**

- An understanding of why data needs to be collected
- Experience of simple tally charts
- Some idea about different types of graphs
- Experience of inequality notation and signs

**OBJECTIVES**

By the end of the module the student should be able to:

- Specify the problem and plan (3.1)
- Decide what data to collect and what statistical analysis is needed (3.1)
- Collect data from a variety of suitable primary and secondary sources (3.1)
- Use suitable data collection techniques (3.1)
- Process and represent the data (3.1)
- Interpret and discuss the data (3.1)
- Understand how sources of data may be biased (3.4)
- Identify which primary data they need to collect and in what format, including grouped data (3.3)
- Consider fairness (3.3)
- Design a question for a questionnaire (3.3)
- Criticise questions for a questionnaire (3.3)
- Design and use data-collection sheets for grouped, discrete and continuous data (3.2)
- Collect data using various methods (3.2)
- Sort, classify and tabulate data and discrete or continuous quantitative data (3.1, 3.2)
- Group discrete and continuous data into class intervals of equal width (3.2)
- Extract data from lists and tables (3.5, 3.1)
- Design and use two-way tables for discrete and grouped data (3.5)
- Use information provided to complete a two way table (3.5)

**DIFFERENTIATION & EXTENSION**

Carry out a statistical investigation of their own, including designing an appropriate means of gathering the data  
 Some guidance needs to be given to stop students choosing limited investigations, eg favourite football team

**NOTES**

For Functional Elements activities, it is worth collecting data at different times of the day, eg to compare types of shopper in a centre. Get data from holiday brochures to compare resorts for temp, rainfall and type of visitor  
 Emphasise the differences between primary and secondary data. Mayfield High data can be used as an example of secondary data

Discuss sample size and mention that a census is the whole population. In the UK, the census is every year that ends in '1', so the next census is in 2011

If students are collecting data as a group, then they should use the same procedure  
 Emphasise that continuous data is data that is measured, eg temperature

GCSE Tier: Foundation

Contents: Charts and graphs

- SP g Produce charts and diagrams for various data types  
 SP i Interpret a wide range of graphs and diagrams and draw conclusions  
 SP l Compare distributions and make inferences

**PRIOR KNOWLEDGE:**

An understanding of why data needs to be collected and some idea about different types of graphs

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw: (12.1 – 12.6)
  - Pictograms,
  - Bar charts,
  - Histograms with equal class intervals,
  - Frequency diagrams for grouped discrete data,
  - Line graphs,
  - Frequency polygons for grouped data,
  - Composite bar charts
  - Comparative and dual bar charts
- Interpret: (16.4, 25.2, 12.6)
  - composite bar charts
  - stem and leaf diagrams
  - scatter graphs
  - frequency polygons
- From pictograms, bar charts, line graphs and histograms with equal class intervals: (12.1, 12.3 – 12.5)
  - read off frequency values
  - calculate total population
  - find greatest and least values
- Recognise simple patterns and characteristic relationships in bar charts, line graphs and frequency polygons (12.3 – 12.6, 25.1)
- Use comparative bar charts to compare distributions (12.4)

**DIFFERENTIATION & EXTENSION**

Carry out a statistical investigation of their own and use an appropriate means of displaying the results

Use a spreadsheet to draw different types of graphs

Collect examples of charts and graphs in the media which have been misused, and discuss the implications

**NOTES**

Reiterate that clear presentation with axes correctly labelled is important, and to use a ruler to draw straight lines

Stem and leaf diagrams must have a key

Show how to find the median and the mode from a stem and leaf diagram

Make comparisons between previously collected data

Encourage student to work in groups and present their charts (useful display material for classrooms/corridors)

Use Excel Graph wizard

Consider Functional Elements by comparing rainfall charts, distributions of ages in cinemas etc

**Module** 8

**Time: 3 – 5 hours**

**GCSE Tier: Foundation**

**Contents: Symmetry, Similarity and Congruence**

GM e Recognise reflection and rotation symmetry of 2-D shapes

GM f Understand congruence and similarity

**PRIOR KNOWLEDGE:**

Basic idea of shape

**OBJECTIVES**

By the end of the module the student should be able to:

- Recognise reflection symmetry of 2-D shapes (6.5)
- Understand the concept of line symmetry and be able to identify and draw lines of symmetry on a shape (6.5)
- Draw and complete diagrams with a given number of lines of symmetry (6.5)
- Recognise rotation symmetry of 2-D shapes (6.6)
- Understand the concept of rotational symmetry (6.6)
- Be able to identify the order of rotational symmetry of a 2-D shape (6.6)
- Draw or complete diagrams with a given order of rotational symmetry (6.6)
- Understand congruence (6.3)
- Identify shapes which are congruent (6.3)
- Understand similarity (6.3)
- Identify shapes which are similar, including all circles or all regular polygons with equal number of sides (6.3)
- Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size (6.3)

**DIFFERENTIATION & EXTENSION**

Investigate Rangoli Patterns, which is a good source of display work

Ask students to find their own examples of symmetry, similarity and congruence in real-life

**NOTES**

Equations of lines of symmetry are covered later in course

Reinforce accurate drawing skills and measurement

Use tracing paper or mirrors to assist with symmetry questions

GCSE Tier: Foundation

Contents: Types of number

N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Least Common Multiple (LCM), prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers

**PRIOR KNOWLEDGE:**

Number complements to 10 and multiplication/division facts  
 Recognise basic number patterns  
 Experience of classifying integers

**OBJECTIVES**

By the end of the module the student should be able to:

- Recognise even and odd numbers (1.10)
- Identify factors, multiples and prime numbers (1.10)
- Find the prime factor decomposition of positive integers (1.10 – 1.11)
- Find the common factors and common multiples of two or three numbers (1.11)
- Find the Lowest common multiple (LCM) and Highest common factor (HCF) of two or three numbers (1.11)
- Recall integer squares up to  $15 \times 15$  and the corresponding square roots (1.12, 5.5)
- Recall the cubes of 2, 3, 4, 5 and 10 (1.12, 5.5)
- Find the value of a square or square root (1.12, 5.5)
- Use index notation for squares and cubes (9.1)
- Use index notation for powers of 10 (9.1 – 9.2)
- Find the value of calculations, which include indices (9.1 – 9.2)

**DIFFERENTIATION & EXTENSION**

Calculator exercise to check factors of larger numbers  
 Further work on indices to include negative and/or fractional indices  
 Use prime factors to find LCM  
 Use a number square to find primes (sieve of Eratosthenes)  
 Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

**NOTES**

All of the work in this module can be easily reinforced by using it as ‘starters’ or ‘plenaries’  
 Calculators should be used only when appropriate  
 There are plenty of investigative work using squares like ‘half time’ scores  
 For extension, work could introduce simple ideas on standard form

Module 10

Time: 3 – 5 hours

GCSE Tier: Foundation

Contents: Introduction to algebra

- A a Distinguish the different roles played by letter symbols in algebra, using the correct notation  
A b Distinguish in meaning between the words ‘equation’, ‘formula’ and ‘expression’  
A c Manipulate algebraic expressions by collecting like terms

**PRIOR KNOWLEDGE:**

- Experience of using a letter to represent a number  
Ability to use negative numbers with the four operations

**OBJECTIVES**

By the end of the module the student should be able to:

- Use notation and symbols correctly (4.1)
- Write an expression (4.1, 4.2)
- Simplify algebraic expressions in one or more like terms, by adding and subtracting (4.3)
- Understand the difference between the word ‘equation’, ‘formula’, and ‘expression’ (4.8)
- Multiply and divide with variables and numbers (4.4, 4.5)

**DIFFERENTIATION & EXTENSION**

Look at patterns in games like ‘frogs’, eg Total moves =  $R \times G + R + G$

Look at methods to understand expressions, eg there are ‘ $b$ ’ boys and ‘ $g$ ’ girls in a class, what is the total ‘ $t$ ’ number of students in the class

Further work, such as collecting like terms involving negative terms, collecting terms where each term may consist of more than one letter eg  $3ab + 4ab$

**NOTES**

- Emphasise correct use of symbolic notation, eg  $3x$  rather than  $3 \times x$   
Present all work neatly and use the appropriate algebraic vocabulary

**Module 11**

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Constructions**

GM v Use straight edge and a pair of compasses to carry out constructions

GM w Construct loci

**PRIOR KNOWLEDGE:**

Knowledge of types of triangle

Knowledge of the difference between a line and a region

**OBJECTIVES**

By the end of the module the student should be able to:

- Use straight edge and a pair of compasses to do standard constructions such as **(18.1)**
  - Construct a triangle
  - Construct an equilateral triangle
  - Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not
  - Construct the perpendicular bisector of a given line
  - Construct the perpendicular from a point to a line
  - Construct the bisector of a given angle
  - Construct angles of  $60^\circ$ ,  $90^\circ$ ,  $30^\circ$ ,  $45^\circ$
  - Draw parallel lines
  - Construct diagrams of everyday 2 - D situations involving rectangles, triangles, perpendicular and parallel lines
- Draw and construct diagrams from given instructions **(18.2, 18.3)**
  - A region bounded by a circle and an intersecting line
  - A given distance from a point and a given distance from a line
  - Equal distances from 2 points or 2 line segments
  - Regions which may be defined by ‘nearer to’ or ‘greater than’
  - Find and describe regions satisfying a combination of loci

**DIFFERENTIATION & EXTENSION**

Try to do this module as practically as possible using real life situations eg horses tethered to ropes, mobile phone masts etc

Use the internet to source ideas for this module

Use loci problems that require a combination of loci

**NOTES**

All constructions should be presently neatly and accurately

A sturdy pair of compasses is essential

Construction lines should not be erased as they carry valuable method marks

All lines should be correct to within 2 mm and angles correct to  $2^\circ$

**Module 12**

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Patterns and sequences**

A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence

A j Use linear expressions to describe the  $n^{\text{th}}$  term of an arithmetic sequence

**PRIOR KNOWLEDGE:**

- Know about odd and even numbers
- Recognise simple number patterns, eg 1, 3, 5, ...
- Writing simple rules algebraically
- Raise numbers to positive whole number powers

**OBJECTIVES**

By the end of the module the student should be able to:

- Generate simple sequences of odd or even numbers (13.1)
- Find the missing numbers in a number pattern or sequence (13.1 – 13.3)
- Find the  $n^{\text{th}}$  term of a number sequence (13.2, 13.3)
- Use the  $n^{\text{th}}$  number of an arithmetic sequence (13.3)
- Find whether a number is part of a given sequence (13.4)
- Continue a sequence derived from diagrams (13.1)
- Use a calculator to produce a sequence of numbers (13.1 – 13.3)

**DIFFERENTIATION & EXTENSION**

- Match-stick problems
- Use practical real life examples like ‘flower beds’
- Sequences of triangle numbers, Fibonacci numbers etc
- Extend to quadratic sequences whose  $n^{\text{th}}$  term is  $an^2 + b$  and link to square numbers

**NOTES**

- Emphasise good use of notation  $3n$  means  $3 \times n$
- When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the  $n^{\text{th}}$  term

GCSE Tier: Foundation

Contents: Properties of quadrilaterals and parallel lines

- GM d Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
- GM b Understand and use the angle properties of parallel lines, and quadrilaterals
- GM r Understand and use bearings

**PRIOR KNOWLEDGE:**

- Know that angles in a triangle add up to  $180^\circ$
- Know that angles at a point on a straight line sum to  $180^\circ$
- Know that a right angle =  $90^\circ$

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall the properties and definitions of special types of quadrilaterals, including symmetry properties (6.2)
- List the properties of each, or identify (name) a given shape (6.2)
- Draw sketches of shapes (19.1)
- Name all quadrilaterals that have a specific property (6.2)
- Identify quadrilaterals from everyday usage (19.1)
- Classify quadrilaterals by their geometric properties (6.2)
- Understand and use the angle properties of parallel lines (7.5)
- Mark parallel lines on a diagram (7.5)
- Find missing angles using properties of corresponding and alternate angles (7.6)
- Understand and use the angle properties of quadrilaterals (7.1)
- Use the fact that angle sum of a quadrilateral is  $360^\circ$  (7.1)
- Give reasons for angle calculations (Ch 7)
- Use bearings to specify direction (7.8)
- Use three figure-bearings to specify direction (7.8)
- Mark on a diagram the position of point  $B$  given its bearing from the point  $A$  (7.8)
- Give a bearing between the points on a map or scaled plan (7.8)
- Given the bearing of point  $A$  from point  $B$ , work out the bearing of  $B$  from  $A$  (7.8)

**DIFFERENTIATION & EXTENSION**

- Practical activities help with the understanding of the properties and proofs – games like ‘Guess who I am?’
- Use the angle properties of triangles to find missing angles in combinations of triangles
- Explore other properties in triangles, quadrilaterals and parallel lines

**NOTES**

- All diagrams should be presented neatly and accurately
- Students should have plenty of practice drawing examples to illustrate the properties of various shapes
- For bearings and scaled drawings, angles should be correct to  $2^\circ$  and lines accurate to 2 mm

**Module 14**

**Time: 6 – 8 hours**

**GCSE Tier: Foundation**

**Contents: Fractions**

N h	Understand equivalent fractions, simplifying a fraction by cancelling all common factors
N i	Add and subtract fractions
N b	Order rational numbers
N j	Use decimal notation and understand that decimals and fractions are equivalent
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals

**PRIOR KNOWLEDGE:**

- Multiplication facts
- Ability to find common factors
- A basic understanding of fractions as being ‘parts of a whole unit’
- Use of a calculator with fractions

**OBJECTIVES**

By the end of the module the student should be able to:

- Visualise a fraction diagrammatically **(8.1)**
- Understand a fraction as part of a whole **(8.1)**
- Recognise and write fractions in everyday situations **(8.1)**
- Find fractions of amounts **(8.5)**
- Write a fraction in its simplest form and recognise equivalent fractions **(8.2)**
- Compare the sizes of fractions using a common denominator **(8.3)**
- Add and subtract fractions by using a common denominator **(8.7)**
- Write an improper fraction as a mixed fraction **(8.4)**
- Convert between fractions and decimals **(8.8)**

**DIFFERENTIATION & EXTENSION**

- Careful differentiation is essential as this topic is dependent on the student’s ability
- Relate simple fractions to percentages and vice versa
- Work with improper fractions and mixed numbers, eg divide 5 pizza’s between 3 people
- Solve word problems involving fractions and in real life problems, eg finding the perimeter using fractional values
- Link fractions with probability questions

**NOTES**

- Regular revision of fractions is essential
- Demonstrate how to use the fraction button on a calculator, in order be able to check solutions
- Use real-life examples whenever possible

**Module 15**

**Time: 3 – 4 hours**

**GCSE Tier: Foundation**

**Contents: Pie charts**

SP g Draw and produce pie charts  
SP i Interpret pie charts  
SP l Compare distributions and make inferences

**PRIOR KNOWLEDGE:**

Measuring and drawing angles  
Fractions of simple quantities

**OBJECTIVES**

By the end of the module the student should be able to:

- Represent data in a pie chart (12.2)
- Interpret data in a pie chart (12.2)
- Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts (12.2)
- From pie charts (12.2)
  - work out amounts for each category
  - calculate the total population
  - find greatest and least values

**DIFFERENTIATION & EXTENSION**

Use this module to refresh memories on frequency and tally tables  
Practise the ability to divide by 20, 30, 40, 60 etc  
This can be delivered as a practical module that could lead to wall display- remind about of bias, eg only asking their friends which band they like  
Compare pie charts for, eg boys and girls, to identify similarities and differences  
Ask students to combine two pie charts

**NOTES**

Angles for pie charts should be accurate to within 2°

**Module 16**

**Time: 3 – 5 hours**

**GCSE Tier: Foundation**

**Contents: Fractions, Decimals and Percentages**

- N h Understand equivalent fractions in the context of ‘hundredths’  
N l Understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions  
N o Interpret fractions, decimals and percentages as operators  
N u Approximate to a specified or appropriate degree of accuracy  
N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

- Four operations of number
- The concepts of a fraction and a decimal
- Number complements to 10 and multiplication tables
- Awareness that percentages are used in everyday life

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths **(19.1)**
- Convert between fractions decimals and percentages **(19.1)**
- Write one number as a percentage of another number **(19.4)**
- Calculate the percentage of a given amount **(19.2, 19.3)**

**DIFFERENTIATION & EXTENSION**

Consider fractional percentages of amounts eg  $12.5\% = 0.125 = \frac{1}{8}$

Consider percentages which convert to recurring decimals (eg  $33\frac{1}{3}\%$ ), and situations which lead to percentages of more than 100%

Use fraction, decimal and percentage dominos or follow me cards.

Investigate into the many uses made of percentages, particularly in the media

Practise the ability to convert between different forms

**NOTES**

Use Functional Elements questions using fractions, eg  $\frac{1}{4}$  off the list price when comparing different sale prices

Keep using non-calculator methods, eg start with 10%, then 1% in order to required percentages

**Module** 17

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Applications of percentages**

N m Use percentages  
N o Interpret fractions, decimals and percentages as operators  
N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

Four operations of number  
The concepts of a fraction and a decimal  
Number complements to 10 and multiplication tables  
Awareness that percentages are used in everyday life

**OBJECTIVES**

By the end of the module the student should be able to:

- Use percentages to solve problems **(Ch 19)**
- Convert between fractions, decimals and percentages to find percentage change **(19.1)**
- Find a percentage of a quantity in order to increase or decrease **(19.3)**
- Use percentages in real-life situations **(19.2)**
  - price after VAT
  - profit or loss
  - simple interest
  - income tax calculations
  - annual rate of inflation

**DIFFERENTIATION & EXTENSION**

Use a mixture of calculator and non-calculator methods  
Use ideas for wall display, students make up their own poster to explain say a holiday reduction  
Use functional skills questions to look at questions in context  
Combine multipliers to simplify a series of percentage changes  
Problems which lead to the necessity of rounding to the nearest penny, eg real-life contexts  
Investigate comparisons between simple and compound interest calculations

**NOTES**

Use plenty of practical examples that can be linked to Functional Elements, eg VAT calculations

**Module** 18

**Time: 3 –5 hours**

**GCSE Tier: Foundation**

**Contents: Algebra using powers and brackets**

N f Use the index laws for multiplication and division of integer powers

A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors

**PRIOR KNOWLEDGE:**

Squares and cubes

Experience of using a letter to represent a number

Ability to use negative numbers with the four operations

**OBJECTIVES**

By the end of the module the student should be able to:

- Use index laws for multiplication and division of integer powers, and of powers of a power (9.2)
- Find the value of calculations using indices without using a calculator (9.2, 28.2)
- Multiply a single algebraic term over a bracket (9.5, 4.6)
- Write expressions using squares and cubes (9.3, 4.4)
- Use simple instances of index laws (9.3)
- Factorise algebraic expressions by taking out common factors (9.6, 4.7)

**DIFFERENTIATION & EXTENSION**

Use various investigations leading to generalisations, eg:

Indices – cell growth, paper folding

Brackets – pond borders  $4n + 4$  or  $4(n + 1)$

Football league matches  $n^2 - n$  or  $n(n - 1)$

**NOTES**

Use everyday examples that lead to generalisations

**Module 19**

**Time: 5 – 7 hours**

**GCSE Tier: Foundation**

**Contents: Ratio and proportion**

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N t	Divide a quantity in a given ratio
GM m	Use and interpret maps and scale drawings
N q	Understand and use number operations and inverse operations

**PRIOR KNOWLEDGE:**

- Using the four operations
- Ability to recognise common factors
- Knowledge of fractions

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand what is meant by ratio (24.1)
- Write a ratio in its simplest form; and find an equivalent ratio (24.1)
- Solve a ratio problem in context, eg recipes (24.2)
- Share a quantity in a given ratio (24.3)
- Understand and use examples in direct proportion (24.4)
- Interpret map/model scales as a ratio (7.9)
- Solve problems involving money conversions, eg £'s to Euros etc (22.2)

**DIFFERENTIATION & EXTENSION**

- Consider maps: draw a plan of the school
- Further problems involving scale drawing, eg find the real distance in metres between two points on 1 : 40000 map
- Plan a housing estate with variety of different sized houses
- Currency calculations using foreign exchange rates
- Harder examples involving multi-stage problems
- Link ratios and proportion to Functional Elements, eg investigate the proportion of different metals in alloys, the ingredients needed for recipes for fewer or more people, mixing cement, planting forests, comparing prices of goods here and abroad, Best buy type questions

**NOTES**

Students often find ratios with 3 parts difficult

**Module**            **20**

**Time: 5 – 7 hours**

**GCSE Tier:**      **Foundation**

**Contents:**        **Linear equations and inequalities**

- A d            Set up and solve simple equations  
N q            Understand and use number operations and the relationships between them including inverse operations and the hierarchy of operations  
A g            Solve linear inequalities in one variable and represent the solution set on a number line

**PRIOR KNOWLEDGE:**

- Experience of finding missing numbers in calculations
- The idea that some operations are ‘opposite’ to each other
- An understanding of balancing
- Experience of using letters to represent quantities
- Be able to draw a number line
- An understanding of fractions and negative numbers

**OBJECTIVES**

By the end of the module the student should be able to:

- Set up simple equations (21.1)
- Rearrange simple equations (21.1, 21.2)
- Solve simple equations (21.1, 21.2)
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation (21.3 – 21.6)
- Solve linear equations which include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution (21.3 – 21.6)
- Solve linear equations in one unknown, with integer or fractional coefficients (21.3 – 21.6)
- Use linear equations to solve word problems (21.7)
- Solve simple linear inequalities in one variable, and represent the solution set on a number line (21.10)
- Use the correct notation to show inclusive and exclusive inequalities (21.11)

**DIFFERENTIATION & EXTENSION**

- Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)
- Solve equations where manipulation of fractions (including negative fractions) is required

**NOTES**

- Remind students about work on patterns and sequences that have linear results
- Students need to realise that not all equations can be solved by ‘trial and improvement’ or by observation. The use of a formal method of solving equations is very important
- Remind students of the need to set their work out clearly, keeping the equal signs in line

**Module**            **21**

**Time: 6 – 8 hours**

**GCSE Tier:**      **Foundation**

**Contents:**        **Perimeter and area**

GM x            Calculate perimeters and areas of shapes made from triangles and rectangles  
GM n            Understand the effect of enlargement for perimeter and area of shapes  
GM p            Convert between units and area measures

**PRIOR KNOWLEDGE:**

Names of triangles, quadrilaterals  
Knowledge of the properties of rectangles, parallelograms and triangles  
Concept of perimeter and area  
Units of measurement  
Four operations of number

**OBJECTIVES**

By the end of the module the student should be able to:

- Measure shapes to find perimeters and areas **(14.1, 14.3)**
- Find the perimeter of rectangles and triangles **(14.1)**
- Find the perimeter of compound shapes **(14.1)**
- Find the area of a rectangle and triangle **(14.2, 14.3)**
- Recall and use the formulae for the area of a triangle, rectangle and a parallelogram **(14.3)**
- Calculate areas of compound shapes made from triangles and rectangles **(14.2)**
- Find the area of a trapezium by using the formula **(14.3)**
- Solve a range of problems involving areas including cost of carpet type questions **(14.4)**
- Convert between units of area **(20.7)**

**DIFFERENTIATION & EXTENSION**

Further problems involving combinations of shapes  
Use practical examples from functional papers on topics such as turfing a garden, carpeting a room, laying carpet tiles on a floor  
Perimeter questions could use skirting board, wallpaper, planting a border of a garden

**NOTES**

Discuss the correct use of language and units, particularly when method marks are for the correct unit of measure  
Ensure that students can distinguish between perimeter and area  
Practical examples help to clarify the concepts, eg floor tiles etc

**Module**            **22**

**Time: 3 – 5 hours**

**GCSE Tier:**      **Foundation**

**Contents:**        **3-D shapes**

GM k            Use 2-D representations of 3-D shapes

GM x            Calculate the surface area of a 3-D shape

**PRIOR KNOWLEDGE:**

The names of standard 2-D and 3-D shapes

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone            **(20.2)**
- Know the terms face, edge and vertex            **(20.1)**
- Use 2-D representations of 3-D shapes            **(20.1 – 20.3)**
- Use isometric grids            **(20.2)**
- Draw nets and show how they fold to make a 3-D solid            **(20.2)**
- Understand and draw front and side elevations and plans of shapes made from simple solids            **(20.3)**
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3-D solid            **(20.3)**

**DIFFERENTIATION & EXTENSION**

Make solids using equipment such as clixi or multi-link

Draw shapes made from multi-link on isometric paper

Build shapes from cubes that are represented in 2-D

Euler's theorem

A useful topic for a wall display-pupils tend to like to draw 3-D shapes and add interest by using a mixture of colours in the elevations

**NOTES**

Accurate drawing skills need to be reinforced

Some students find visualising 3-D object difficult, so using simple models will help

**Module** 23

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Real-life graphs**

A r Construct linear functions from real-life problems and plotting their corresponding graphs

A s Discuss, plot and interpret graphs (which may be non-linear) modelling real situations

**PRIOR KNOWLEDGE:**

Experience at plotting points in all quadrants

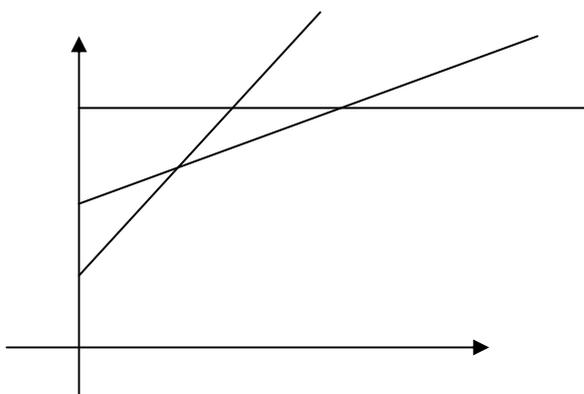
Experience at labelling axes and reading scales

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw graphs representing ‘real’ examples like filling a bath/containers (22.1 – 22.3)
- Interpret and draw linear graphs, including conversion graphs, fuel bills etc (22.1 – 22.3)
- Solve problems relating to mobile phone bills with fixed charge and price per unit (22.1 – 22.3)
- Interpret non-linear graphs like the number of cans in a vending machine at times throughout the day (22.1 – 22.3)

**DIFFERENTIATION & EXTENSION**



Use open ended questions that test student awareness of what intersections mean, eg mobile phone bills

Use spreadsheets to generate straight-line graphs and pose questions about gradient of lines

Use ICT packages or graphical calculators to draw straight line graphs and quadratic graphs

**NOTES**

Clear presentation is important with axes clearly labelled

Students need to be able to recognise linear graphs and also be able to recognise when their graph is incorrect

Link graphs and relationships in other subject areas, eg science, geography

Students should have plenty of practice interpreting linear graphs for Functional Elements problems

GCSE Tier: Foundation

Contents: Straight line graphs

A1 Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients

**PRIOR KNOWLEDGE:**

Experience at plotting points in all quadrants

**OBJECTIVES**

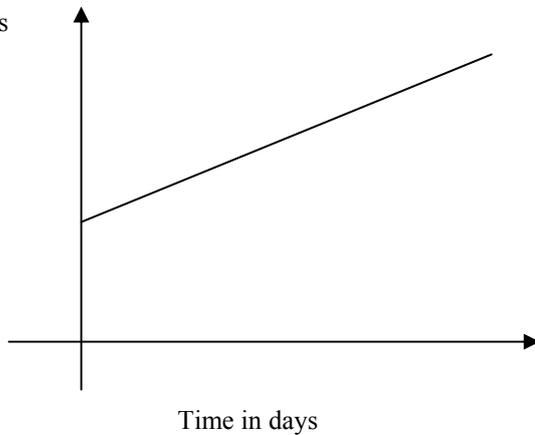
By the end of the module the student should be able to:

- Draw, label and put suitable scales on axes (15.1, Ch 15)
- Recognise (when values are given for  $m$  and  $c$ ) that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane (15.5, 15.6)
- Plot and draw graphs of functions in which  $y$  is given explicitly in terms of  $x$ , or implicitly, and by using tables of values (15.5, 15.6)
- Plot and draw graphs of straight lines of the form  $y = mx + c$  (15.5, 15.6)
- Recognise that, in a mathematical context, an equation in the form  $y = mx + c$  is a straight line (15.5, 15.6)
- Find the gradient of a straight line from a graph (15.7)
- Interpret gradients from real life graphs eg for height of bath water vs time, the gradient is the rate of filling (22.1 – 22.3)

**DIFFERENTIATION & EXTENSION**

Plot graphs of the form  $y = mx + c$  where pupil has to generate their own table and set out their own axes  
 Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines  
 Use a graphical calculator or graphical ICT package to draw straight-line graphs  
 Use some examples from the last module to interpret gradient and intercept

Charge in £'s and the



For hire of a skip the intercept is delivery charge and the gradient is the cost per day

Find the equation of a straight line through two points

**NOTES**

Careful annotation should be encouraged. Label the coordinate axes and write the equation of the line on the graph  
 Cover horizontal and vertical line graphs as students often forget these ( $x = c$  and  $y = c$ )  
 Link graphs and relationships in other subject areas, eg science and geography  
 Interpret straight line graphs in functional elements  
 Link conversion graphs to converting metric and imperial units and equivalents

**Module**            **25**

**Time: 4 – 6 hours**

**GCSE Tier:**      **Foundation**

**Contents:**        **Compound measures**

GM s        Understand and use compound measures  
N u        Approximate to specified or appropriate degree of accuracy  
GM p        Convert between speed measures

**PRIOR KNOWLEDGE:**

Knowledge of metric units eg  $1\text{ m} = 100\text{ cm}$   
Know that  $1\text{ hour} = 60\text{ mins}$ ,  $1\text{ min} = 60\text{ seconds}$   
Experience of multiply by powers of 10, eg  $100 \times 100 = 10\ 000$

**OBJECTIVES**

By the end of the module the student should be able to:

- Use the relationship between distance, speed and time to solve problems **(11.5)**
- Convert between metric units of speed, eg km/h to m/s **(11.5)**

**DIFFERENTIATION & EXTENSION**

Convert imperial units to metric units, eg mph into km/h which would remind students that  $5\text{ miles} = 8\text{ km}$   
Ask students to convert a 100m time of 10 secs into mph  
Use the internet and/or reference books to find weights, volumes and heights of large structures; such as prominent buildings, aeroplanes and ships

**NOTES**

Measurement is a practical activity  
All working out should be shown with multiplication or division by powers of 10  
Use the distance/speed/time triangle (i.e. Drink Some Tea)  
Use Functional Elements for practice questions for this topic area, eg Best Buys  
Link to their work drawing and interpreting

**Module 26**

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Timetables and Distance/ time Graphs**

GM o Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements  
SP e Extract data from printed tables and lists  
A s Interpret graphs (which may be non-linear) modelling real-life situations

**PRIOR KNOWLEDGE:**

Knowledge of metric units eg 1 m = 100 cm  
Know that 1 hour = 60 mins, 1 min = 60 seconds  
Know how to find speed  
Know how to read scales, draw and interpret graphs

**OBJECTIVES**

By the end of the module the student should be able to:

- Convert between 12-hour and 24-hour hour clock times (11.2)
- Read bus and train timetables and plan journeys (11.2)
- Draw distance time graphs (22.3)
- Interpret distance time graphs and solve problems (22.3)

**DIFFERENTIATION & EXTENSION**

Draw and interpret non linear curves like 400 m running track  
Make up a graph and supply the commentary for it  
Ask students to plan and cost a holiday from a brochure

**NOTES**

Clear presentation with axes labelled correctly is important  
Interpret straight line graphs for Functional Elements problems

**Module** 27

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Volume**

- GM aa Calculate volumes of right prisms and shapes made from cubes and cuboids  
GM n Understand the effect of enlargement for perimeter, area and volume of shapes and solids  
GM p Converting between volume measures, including cubic centimetres and cubic metres

**PRIOR KNOWLEDGE:**

- Concept of volume
- Concept of prism
- Experience of constructing cubes or cuboids from multi link

**OBJECTIVES**

By the end of the module the student should be able to:

- Find volumes of shapes by counting cubes (20.4)
- Recall and use formulae for the volume of cubes and cuboids (20.4)
- Calculate the volumes of right prisms and shapes made from cubes and cuboids (20.4, 20.6 enlargement)
- Convert between units of volume (20.7)

**DIFFERENTIATION & EXTENSION**

Look at 'practical' examples with fish tanks/ filling containers, find the number of small boxes fitting into a large box  
Further problems involving a combination of shapes  
Cylinders are left until later in the course

**NOTES**

Discuss the correct use of language and units. Remind students that there is often a mark attached to writing down the correct unit  
Use practical problems to enable the students to understand the difference between perimeter, area and volume  
Use Functional Elements problems, eg floor tiles, optimisation type questions etc

GCSE Tier: Foundation

Contents: Probability

SP m	Understand and use the vocabulary of probability and probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

**PRIOR KNOWLEDGE:**

Elementary fractions decimals and percentages  
Ability to read from a two-way table

**OBJECTIVES**

By the end of the module the student should be able to:

- Distinguish between events which are: impossible, unlikely, even chance, likely, and certain to occur (26.1)
- Mark events and/or probabilities on a probability scale of 0 to 1 (26.1)
- Write probabilities in words, fractions, decimals and percentages (26.1, 26.2)
- Find the probability of an event happening using theoretical probability (26.2, 26.3)
- Find the probability of an event happening using relative frequency (26.5)
- Estimate the number of times an event will occur, given the probability and the number of trials (26.7)
- List all outcomes for single events systematically (26.4)
- List all outcomes for two successive events systematically (26.4)
- Use and draw sample space diagrams (26.4)
- Add simple probabilities (26.3)
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 (26.3)
- Use  $1 - p$  as the probability of an event not occurring where  $p$  is the probability of the event occurring (26.3)
- Find a missing probability from a list or table (26.3)
- Compare experimental data and theoretical probabilities (26.5, 26.7)
- Compare relative frequencies from samples of different sizes (26.5)

**DIFFERENTIATION & EXTENSION**

Use this as an opportunity for practical work  
Experiments with dice and spinners  
Show sample space for outcomes of throwing two dice (36 outcomes)  
Use 'the horse race'/drawing pins/let students make their own biased dice and find experimental probability

**NOTES**

Students should express probabilities as fractions, percentages or decimals  
Probabilities written as fractions don't need to be cancelled to their simplest form

**Module** 29

**Time: 6 – 8 hours**

**GCSE Tier: Foundation**

**Contents: Formulae**

A f Derive a formula, substitute numbers into a formula and change the subject of a formula

**PRIOR KNOWLEDGE:**

Understanding of the mathematical meaning of the words; expression, simplifying, formulae and equation  
Experience of using letters to represent quantities  
Substituting into simple expressions using words  
Using brackets in numerical calculations and removing brackets in simple algebraic expressions

**OBJECTIVES**

By the end of the module the student should be able to:

- Derive a simple formula, including those with squares, cubes and roots (28.4)
- Use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols (28.3, 28.4)
- Substitute numbers into a formula (4.9, 28.2 – 28.5)
- Substitute positive and negative numbers into expressions such as  $3x^2 + 4$  and  $2x^3$  (28.2 – 28.5)
- Change the subject of a formula (28.6)
- Find the solution to a problem by writing an equation and solving it (28.4)

**DIFFERENTIATION & EXTENSION**

Use negative numbers in formulae involving indices  
Various investigations leading to generalisations, eg the painted cube, Frogs, Pond Borders  
Relate to topic on graphs of real life functions  
More complex changing the subject, moving onto higher tier work  
Apply changing of the subject to physics formulae, eg speed, density, equations of motion

**NOTES**

Emphasise the need for good algebraic notation  
Show a linear equation first and follow the same steps to rearrange a similarly structured formula  
Link with Functional Elements problems in everyday problems  
Link with formulae for area and volume

**Module 30**

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Angle properties of polygons**

GM c Calculate and use the sums of the interior and exterior angles of polygons

GM v Use straight edge and a pair of compasses to carry out constructions

**PRIOR KNOWLEDGE:**

Angles on straight lines and in simple shapes

**OBJECTIVES**

By the end of the module the student should be able to:

- Calculate and use the sums of the interior angles of polygons (7.2)
- Use geometrical language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons (7.2)
- Know, or work out, the relationship between the number of sides of a polygon and the sum of its interior angles (7.2)
- Know that the sum of the exterior angles of any polygon is  $360^\circ$  (7.3)
- Calculate the size of each exterior/interior angle of a regular polygon (7.2, 7.3)
- Construct for example a regular hexagon inside a circle (18.1)
- Understand tessellations of regular and irregular polygons (7.4)
- Tessellate combinations of polygons (7.4)
- Explain why some shapes tessellate and why other shapes do not (7.4)

**DIFFERENTIATION & EXTENSION**

Study Escher drawings (possibly cross curricular with Art).

Ask students to design their own tessellation, and explain why their shapes tessellate

**NOTES**

All diagrams should be neatly presented

Use of tracing paper helps with tessellations

Consider real-life examples of tessellations

GCSE Tier: Foundation

Contents: Transformations

GM1 Describe and transform 2-D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations

**PRIOR KNOWLEDGE:**

Recognition of basic shapes

An understanding of the concept of rotation, reflection and enlargement

Coordinates in four quadrants

Linear equations parallel to the coordinate axes and  $y = \pm x$ **OBJECTIVES**

By the end of the module the student should be able to:

- Describe and transform 2-D shapes using single rotations (23.3)
- Understand that rotations are specified by a centre and an (anticlockwise) angle (23.3)
- Find the centre of rotation (23.3)
- Rotate a shape about the origin, or any other point (23.3)
- Describe and transform 2-D shapes using single reflections (23.4)
- Understand that reflections are specified by a mirror line (23.4)
- Identify the equation of a mirror line of symmetry (23.4)
- Describe and transform 2-D shapes using single translations (23.2)
- Understand that translations are specified by a distance and direction, (using a vector) (23.2)
- Translate a given shape by a vector (23.2)
- Describe and transform 2-D shapes using enlargements by a positive scale factor (23.5)
- Understand that an enlargement is specified by a centre and a scale factor (23.5)
- Scale a shape on a grid (without a centre specified) (23.5)
- Enlarge a given shape using (0, 0) as the centre of enlargement (23.5)
- Enlarge shapes with a centre other than (0, 0) (23.5)
- Find the centre of enlargement (23.5)
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides (23.5)
- Describe and transform 2-D shapes using combined rotations, reflections, translations, or enlargements (23.6)
- Understand that distances and angles are preserved under rotations, reflections and translations, so that any shape is congruent under any of these transformations (23.2 - 23.6)
- Describe a transformation (23.2 - 23.6)

**DIFFERENTIATION & EXTENSION**

Use squared paper to enlarge cartoon characters to make a display

**NOTES**

Emphasise that students should describe the transformations fully

Diagrams should be drawn in pencil

Tracing paper can be useful for rotations

**Module** 32

**Time: 5 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Scatter graphs and correlation**

SP k Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent

SP j Look at data to find patterns and exceptions

**PRIOR KNOWLEDGE:**

Plotting coordinates and scale

An understanding of the concept of a variable

Recognition that a change in one variable can affect another

Linear graphs

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw and produce a scatter graph (25.2)
- Look at data to find patterns and exceptions (25.2 - 25.5)
- Distinguish between positive, negative and zero correlation using lines of best fit (25.3 - 25.4)
- Interpret correlation in terms of the problem (25.3 - 25.4)
- Understand that correlation does not imply causality (25.3)
- Draw lines of best fit by eye and understand what it represents (25.4)
- Use a line of best fit to predict values of one variable given values of the other variable (25.5)

**DIFFERENTIATION & EXTENSION**

Vary the axes required on a scatter graph to suit the ability of the class

Carry out a statistical investigation of their own including; designing an appropriate means of gathering the data, and an appropriate means of displaying the results eg height and length of arm

Use a spreadsheet, or other software, to produce scatter diagrams/lines of best fit

Investigate how the line of best fit is affected by the choice of scales on the axes, eg use car data with age and price of the same make of car

**NOTES**

The line of best fit should pass through the coordinate representing the mean of the data

Label all axes clearly and use a ruler to draw all straight lines

Remind student the line of best fit does not necessarily go through the origin of the graph

**Module** 33

**Time: 6 – 8 hours**

**GCSE Tier: Foundation**

**Contents: Averages and Range**

SP h	Calculate median, mean, range, mode and modal class
SP l	Compare distributions and make inferences
SP u	Use calculators efficiently and effectively, including statistical functions
SP g	Produce ordered stem and leaf diagrams
SP i	Draw conclusions from graphs and diagrams

**PRIOR KNOWLEDGE:**

Mid point of a line segment  
Addition and subtraction  
Different statistical diagrams

**OBJECTIVES**

By the end of the module the student should be able to:

- Calculate the mean, mode, median and range for discrete data **(16.1, 16.3)**
- Calculate the mean, mode, median and range from an ordered stem and leaf diagram **(16.4)**
- Produce an ordered stem and leaf diagram **(16.4)**
- Calculate the modal class and interval containing the median for continuous data **(16.6)**
- Calculate the mean, median and mode from a frequency table **(16.5)**
- Estimate the mean of grouped data using the mid-interval value **(16.7)**
- Compare the mean and range of two distributions **(16.3)**
- Recognise the advantages and disadvantages between measures of average **(16.2)**
- Calculate the mean of a small data set, using the appropriate key on a scientific calculator **(16.1)**

**DIFFERENTIATION & EXTENSION**

Find the mean for grouped continuous data with unequal class intervals  
Collect continuous data and decide on appropriate (equal) class intervals; then find measures of average  
Use the statistical functions on a calculator or a spreadsheet to calculate the mean for continuous data

**NOTES**

Ask class to do their own survey with data collection sheets, eg to find the average number of children per family in the class  
The internet and old coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*, *Mayfield High* data etc

Module 34

Time: 2 – 4 hours

GCSE Tier: Foundation

Contents: Quadratic graphs

A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

Squaring negative numbers

Substituting numbers into algebraic expressions

Plotting points on a coordinate grid

Experience of dealing with algebraic expression with brackets -BIDMAS

**OBJECTIVES**

By the end of the module the student should be able to:

- Substitute values of  $x$  into a quadratic function to find the corresponding values of  $y$  (22.4)
- Draw graphs of quadratic functions (22.4)
- Use quadratic graphs to solve approximate solution of a quadratic equation from the graph of the corresponding quadratic function (22.5)

**DIFFERENTIATION & EXTENSION**

Draw simple cubic and  $\frac{1}{x}$  graphs

Solve simultaneous equations graphically including a quadratic graph and a line

Solve simple projectile problems

**NOTES**

The graphs of quadratic functions should be drawn freehand, and in pencil. Turning the paper often helps.

Squaring negative integers may be a problem for some.

Students will often forget the middle term of the expansion and they will need to be reminded of this

**Module** 35

**Time: 3 – 4 hours**

**GCSE Tier: Foundation**

**Contents: Trial and improvement**

- A h Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- N u Approximate to a specified or appropriate degree of accuracy
- N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

- Substituting numbers into algebraic expressions
- Dealing with decimals on a calculator
- Comparing/ordering decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve algebraic equations involving squares and cubes eg  $x^3 + 3x = 40$  (21.8)
- Solve ‘real life’ problems on areas and volumes eg the length of a rectangular room is 2 m longer than the width. If the area is  $30 \text{ m}^2$ , find the width. (21.8)

**DIFFERENTIATION & EXTENSION**

- Can look at various calculator functions like ‘square root’ and ‘cube root’.
- Introduce by trial and improvement
- Solve functions of the form  $\frac{1}{x} = x^2 - 5$ , and link with changing the subject

**NOTES**

- Students should be encouraged to use their calculator efficiently by using the ‘replay’ or ANS/EXE function keys
- Students to take care when entering negative values to be squared
- Students should write down all the digits on their calculator display and only round the final answer to the required degree of accuracy

**Module 36**

**Time: 4 – 6 hours**

**GCSE Tier: Foundation**

**Contents: Circles**

- GM i Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- GM z Find circumferences and areas
- N u Approximate to a specified or appropriate degree of accuracy
- N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

The ability to substitute numbers into formulae

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall the definition of a circle and identify and draw parts of a circle **(6.4)**
- Draw a circle given its radius or diameter **(6.4)**
- Find circumferences of circles and areas enclosed by circles **(17.1 - 17.2)**
- Recall and use the formulae for the circumference of a circle and the area enclosed by a circle **(17.1)**
- Use  $\pi \approx 3.142$  or use the  $\pi$  button on a calculator **(17.1 - 17.3)**
- Find the perimeters and areas of semicircles and quarter circles **(17.3)**
- Find the surface area and volume of a cylinder **(20.4 - 20.5)**

**DIFFERENTIATION & EXTENSION**

Use more complex 2-D shapes eg (harder) sectors of circles

Approximate  $\pi$  as  $\frac{22}{7}$

Work backwards to find the radius/diameter given the circumference/area

Apply to real life contexts with laps of running tracks and average speeds

Make a label for a can

Harder problems involving multi-stage calculations

Define a circle by using the language of loci

**NOTES**

All working should be clearly and accurately presented

Use a pencil to draw all diagrams

A sturdy pair of compasses is essential

**Module** 37

**Time: 4 – 5 hours**

**GCSE Tier: Foundation**

**Contents: Pythagoras' Theorem**

Gm g Understand, recall and use Pythagoras' theorem in 2-D  
A k Calculate the length of a line segment  
N u Approximate to specified or appropriate degrees of accuracy  
N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

Knowledge of square and square roots  
Knowledge of types of triangle

**OBJECTIVES**

By the end of this module students should be able to

- Understand and recall Pythagoras' Theorem (27.1 - 27.4)
- Use Pythagoras' theorem to find the hypotenuse (27.1)
- Use Pythagoras' theorem to find a side (27.2)
- Use Pythagoras' theorem to find the length of a line segment from a coordinate grid (27.4)
- Apply Pythagoras' theorem to practical situations (27.3 - 27.4)

**DIFFERENTIATION & EXTENSION**

See exemplar question involving times taken to cross a field as oppose to going around the edge.  
Try to find examples with ladders on walls, area of a sloping roof etc  
Introduce 3-D Pythagoras (moving towards Higher Tier)

**NOTES**

A useful mnemonic for remembering Pythagoras' Theorem is; *'Square it, square it, add/subtract it, square root it'*  
Students should not forget to state units for the answers

## oundation course objectives

N b

Order integers

N u

Approximate to specified or appropriate degrees of accuracy

N a

Add, subtract, multiply and divide any integer

N q

Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

N v

Use calculators effectively and efficiently

N a

Add, subtract multiply and divide any number

N j

Use decimal notation and recognise that each terminating decimal is a fraction

N u

Approximate to specified or appropriate degrees of accuracy

A k

Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

# Foundation course objectives (1MA0)

## Number

N a	Add, subtract, multiply and divide any number
N a	Add, subtract, multiply and divide any integer $< 1$
N b	Order integers
N b	Order rational numbers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Least Common Multiple (LCM), prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers
N h	Understand equivalent fractions, simplifying a fraction by cancelling all common factors
N h	Understand equivalent fractions in the context of 'hundredths'
N i	Add and subtract fractions
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N j	Use decimal notation and understand that decimals and fractions are equivalent
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentages
N o	Interpret fractions, decimals and percentages as operators
N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and inverse operations
N q	Understand and use number operations and the relationships between them including inverse operations and the hierarchy of operations
N t	Divide a quantity in a given ratio
N u	Approximate to specified or appropriate degrees of accuracy
N v	Use calculators effectively and efficiently

# Algebra

- A a Distinguish the different roles played by letter symbols in algebra
- A b Distinguish the meaning between the words 'equation', 'formula' and 'expression'
- A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
- A d Set up and solve simple equations
- A f Derive a formula, substitute numbers into a formula and change the subject of a formula
- A g Solve linear inequalities in one variable and represent the numbers on a number line
- A h Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
- A j Use linear expressions to describe the  $n^{\text{th}}$  term of an arithmetic sequence
- A k Calculate the length of a line segment
- A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
- A l Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
- A r Construct linear functions from real-life problems and plotting their corresponding graphs
- A s Discuss plot and interpret graphs that may be non linear that model real situations
- A s Interpret distance time graphs
- A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

## Geometry and Measures

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and vertically opposite angles
GM b	Understand and use the angle properties of triangles
GM b	Understand and use the angle properties of parallel lines and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM e	Recognise reflection and rotation symmetry of 2-D shapes
GM f	Understand congruence and similarity
Gm g	Understand, recall and use Pythagoras' theorem in 2-D
GM i	Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM k	Use 2-D representations of 3-D shapes
GM l	Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations
GM m	Use and interpret maps and scale drawings
GM n	Understand the effect of enlargement for perimeter, area and volume of shapes and solids
GM o	Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
GM o	Use correct notation for time 12- and 24- hour clock
GM p	Convert measurements from one unit to another
GM p	Convert between units and area measures
GM p	Convert between speed measures
GM p	Convert between volume measures, including cubic centimetres and cubic metres
GM r	Understand and use bearings
GM s	Understand and use compound measures
GM t	Measure and draw lines and angles
GM w	Construct loci
GM u	Draw triangles and other 2-D shapes using a ruler and protractor
GM v	Use straight edge and a pair of compasses to carry out constructions
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles
GM x	Calculate the surface area of a 3-D shape
GM z	Find circumferences and areas
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids

## Statistics and Probability

- SP a Understand and use statistical problem solving process/handling data cycle
- SP a Plan journeys
- SP b Identify possible sources of bias
- SP c Design an experiment or survey
- SP d Design data-collection sheets distinguishing between different types of data
- SP e Extract data from printed tables and lists
- SP e Read timetables
- SP f Design and use two-way tables for discrete and grouped data
- SP g Produce charts and diagrams for various data types
- SP g Draw and produce pie charts
- SP g Produce ordered stem and leaf diagrams
- SP h Calculate median, mean, range, mode and modal class
- SP i Interpret pie charts
- SP i Interpret a wide range of graphs and diagrams and draw conclusions
- SP i Draw conclusions from diagrams
- SP j Look at data to find patterns and exceptions
- SP k Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
- SP l Compare distributions and make inferences
- SP m Understand and use the vocabulary of probability and probability scale
- SP n Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
- SP o List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
- SP p Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- SP s Compare experimental data and theoretical probabilities
- SP t Understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
- SP u Use calculators efficiently and effectively, including statistical functions

# **GCSE Mathematics A (1MA0)**

**Higher  
Tier**

**Linear  
Scheme of Work**



# Higher course overview

The table below shows an overview of modules in the Higher tier scheme of work.

Teachers should be aware that the estimated teaching hours are **approximate** and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Integers and decimals	5
2	Coordinates	3
3	Fractions	5
4	Algebra	7
5	Shape and angle	6
6	Collecting data	4
7	Displaying data	7
8	Construction and loci	5
9	Types of number	7
10	Patterns and sequences	4
11	2-D and 3-D shapes	4
12	Perimeter and Area	7
13	Fractions, Decimals and Percentages	8
14	Formulae and Linear Equations	7
15	Linear Graphs	5
16	Simultaneous Equations	4
17	Probability	7
18	Ratio and Scale	7
19	Averages and Range	8
20	Pythagoras and Trigonometry	8
21	Trial and Improvement	4
22	Surface Area and Volume	7
23	Compound Measures	7
24	Transformations	6
25	Similarity and Congruence	5
26	Quadratic functions, graphs and equations	7
27	Index notation and surds	6
28	Circle theorems	4
29	Sine and cosine rules	5
30	Vectors	5
31	Further graphs and functions	5
32	Transformations of graphs	4
		183 HOURS





## NOTES

The expectation for most students doing Higher tier is that some of this material can be delivered or reinforced during other topics. For example, rounding with significant figures could be done with trigonometry

Present all working clearly with decimal points in line; emphasise that all working is to be shown

For non-calculator methods, make sure that remainders and carrying are shown

Amounts of money should always be rounded to the nearest penny where necessary.

It is essential to ensure the students are absolutely clear about the difference between significant figures and decimal places

Extend to multiplication of decimals and/or long division of integers

Try different methods from the traditional ones, eg Russian or Chinese methods for multiplication etc

Give lots of Functional Elements examples

**Module**            **2**

**Time: 2 - 4 hours**

**GCSE Tier:**    **Higher**

**Contents:**        **Coordinates**

A k            Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

**PRIOR KNOWLEDGE:**

Directed numbers

**OBJECTIVES**

By the end of the module the student should be able to:

- Use axes and coordinates to specify points in all four quadrants in 2-D and 3-D (12.11, 15.1)
- Identify points with given coordinates (12.11, 15.1)
- Identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants) (15.1)
- Find the coordinates of points identified by geometrical information in 2-D and 3-D (12.11, 15.1)
- Find the coordinates of the midpoint of a line segment,  $AB$ , given the coordinates of  $A$  and  $B$  (15.2)

**DIFFERENTIATION & EXTENSION**

There are some excellent interactive 3-D resources which aid student understanding

**NOTES**

This topic can be delivered simultaneously with the properties of simple 2-D and 3-D shapes

**Module 3**

**Time: 4 – 6 hours**

**GCSE Tier: Higher**

**Contents: Fractions**

- N h Understand equivalent fractions, simplifying a fraction by cancelling all common factors  
N i Add and subtract fractions  
N b Order rational numbers  
N a Multiply and divide fractions  
N v Use a calculator effectively and efficiently

**PRIOR KNOWLEDGE:**

- Multiplication facts
- Ability to find common factors
- A basic understanding of fractions as being ‘parts of a whole unit’
- Use of a calculator with fractions

**OBJECTIVES**

By the end of the module the student should be able to:

- Find equivalent fractions (3.1)
- Compare the sizes of fractions (3.1)
- Write a fraction in its simplest form (3.6)
- Find fractions of an amount (3.6)
- Convert between mixed numbers and improper fractions (3.1 - 3.4)
- Add and subtract fractions (3.2, 3.5)
- Multiply and divide fractions including mixed numbers (3.3 - 3.5)

**DIFFERENTIATION & EXTENSION**

- Could introduce ‘hundredths’ at this stage
- Solve word problems involving fractions
- Improper fractions can be introduced by using real-world examples, eg dividing 5 pizzas amongst 3 people
- Careful differentiation is essential for this topic dependent upon the student’s ability
- Use a calculator to change fractions into decimals and look for patterns
- Work with improper fractions and mixed numbers
- Multiplication and division of fractions to link with probability
- Recognising that every terminating decimal has its fraction with 2 and/or 5 as a common factor in the denominator
- Solve word problems and real-world problems involving fractions, eg Find perimeter using fractional values.
- Introduce algebraic fractions

**NOTES**

- Constant revision of this topic is needed
- Use fraction button on the calculator to check solutions
- Link with Probability calculations using AND and OR Laws
- Use fractions for calculations involving compound units
- Use Functional Elements questions and examples using fractions, eg  $\frac{1}{4}$  off the list price when comparing different sale prices

**Module 4**

**Time: 6 – 8 hours**

**GCSE Tier: Higher**

**Contents: Algebra**

- A a Distinguish the different roles played by letter symbols in algebra, using the correct notation  
A b Distinguish in meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’  
A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplifying rational expressions

**PRIOR KNOWLEDGE:**

Experience of using a letter to represent a number  
Ability to use negative numbers with the four operations  
Recall and use BIDMAS

**OBJECTIVES**

By the end of the module the student should be able to:

- Use notation and symbols correctly (2.1)
- Write an expression (2.1)
- Select an expression/identity/equation/formulae from a list (13.6)
- Manipulate algebraic expressions by collecting like terms (2.2)
- Multiply a single term over a bracket (9.1)
- Factorise algebraic expressions by taking out common factors (9.2)
- Expand the product of two linear expressions (9.3)
- Factorise quadratic expressions using the difference of two squares (9.4)
- Simplify rational expressions by cancelling, adding, subtracting, and multiplying (32.1 - 32.3)

**DIFFERENTIATION & EXTENSION**

This topic can be used as a reminder of the KS3 curriculum and could be introduced via investigative material eg frogs, handshakes, patterns in real life, formulae  
Use examples where generalisation skills are required  
Extend the above ideas to the ‘equation’ of the straight line,  $y = mx + c$   
Look at word equations written in symbolic form, eg.  $F = 2C + 30$  to convert temperature (roughly) and compare with  $F = \frac{9}{5}C + 32$   
Practise factorisation where the factor may involve more than one variable

**NOTES**

There are plenty of old exam papers with matching tables testing knowledge of the ‘Vocabulary of Algebra’ (See Emporium website)

GCSE Tier: Higher

Contents: Shape and Angle

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
GM b	Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
Gm r	Understand and use bearings

**PRIOR KNOWLEDGE:**

An understanding of angle as a measure of turning  
 The ability to use a protractor to measure angles  
 Understanding of the concept of parallel lines

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall and use properties of angles (5.1)
  - angles at a point
  - angles at a point on a straight line
  - perpendicular lines
  - vertically opposite angles
- Understand and use the angle properties of parallel lines (5.2)
- Understand, draw and measure bearings (5.8)
- Calculate bearings and solve bearings problems (5.8)
- Mark parallel lines on a diagram (5.2)
- Find missing angles using properties of corresponding and alternate angles (5.2)
- Understand and use the angle properties of quadrilaterals (5.6)
- Give reasons for angle calculations (Chapter 5)
- Explain why the angle sum of a quadrilateral is  $360^\circ$  (5.5)
- Understand the proof that the angle sum of a triangle is  $180^\circ$  (5.5)
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices (5.10)
- Distinguish between scalene, isosceles, equilateral, and right-angled triangles (5.4)
- Understand and use the angle properties of triangles (5.3, 5.4)
- Find a missing angle in a triangle, using the angle sum of a triangle is  $180^\circ$  (5.3, 5.4)
- Recall and use these properties of angles in more complex problems (5.6, 5.9)
- Calculate and use the sums of the interior angles of polygons (5.10)
- Use geometric language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons (5.10)
- Use the angle sums of irregular polygons (5.10)
- Calculate and use the angles of regular polygons (5.10)
- Use the sum of the interior angles of an  $n$  sided polygon (5.10)
- Use the sum of the exterior angles of any polygon is  $360^\circ$  (5.10)
- Use the sum of the interior angle and the exterior angle is  $180^\circ$  (5.10)
- Find the size of each interior angle or the size of each exterior angle or the number of sides of a regular polygon given one of the three pieces of information (5.10)
- Understand tessellations of regular and irregular polygons and combinations of polygons (8.2)
- Explain why some shapes tessellate when other shapes do not (8.2)

**DIFFERENTIATION & EXTENSION**

Use triangles to find the angle sums of polygons  
 Use the angle properties of triangles to find missing angles in combinations of triangles  
 Harder problems involving multi-step calculations  
 Link with symmetry and tessellations

## NOTES

Most of this is KS3, so can be treated as an opportunity for groups of students to present parts of the module to the rest of the class. They could be encouraged to make resources eg follow me cards, puzzles etc for the others to do

Angles in polygons could be investigated algebraically as an investigation

The tessellation can be done as a cross curricular project with Art (Escher)and is good for wall display

Use lots of practical drawing examples to help illustrate properties of various shapes – Group/Displays

Diagrams used in examinations are seldom drawn accurately

Use tracing paper to show which angles in parallel lines are equal

Encourage students to always give their reasons in problems and ‘quote’ the angle fact/theorem used

GCSE Tier: Higher

Contents: Collecting data

SP a	Understand and use statistical problem solving process/handling data cycle
SP b	Identify possible sources of bias
SP c	Design an experiment or survey
SP d	Design data-collection sheets distinguishing between different types of data
SP e	Extract data from printed tables and lists
SP f	Design and use two-way tables for discrete and grouped data

**PRIOR KNOWLEDGE:**

An understanding of why data needs to be collected  
 Experience of simple tally charts  
 Experience of inequality notation

**OBJECTIVES**

By the end of the module the student should be able to:

- Specify the problem and plan (6.1)
- Decide what data to and what statistical analysis is needed (6.2)
- Collect data from a variety of suitable primary and secondary sources (6.4, 6.8)
- Use suitable data collection techniques (6.4)
- Process and represent the data (6.4, 6.6)
- Interpret and discuss the data (6.7)
- Discuss how data relates to a problem, **identify** possible sources of bias and plan to minimise it (6.7)
- Understand how different sample sizes may affect the reliability of conclusions drawn (6.7)
- Identify which primary data they need to collect and in what format, including grouped data (6.4)
- Consider fairness (6.5, 6.7)
- Design a question for a questionnaire by identifying key questions that can be addressed by statistical methods (6.5)
- Criticise questions for a questionnaire (6.5)
- Design an experiment or survey (6.2, 6.5)
- Select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling (6.2, 6.5)
- Use stratified sampling (6.3)
- Design and use data-collection sheets for grouped, discrete and continuous data (6.4)
- Collect data using various methods (6.4, 6.5)
- Sort, classify and tabulate data and discrete or continuous quantitative data (6.1, 6.4, 6.6)
- Group discrete and continuous data into class intervals of equal width (6.4)
- Extract data from lists and tables (6.6, 6.8)
- Design and use two-way tables for discrete and grouped data (6.6)
- Use information provided to complete a two way table (6.6)

**DIFFERENTIATION & EXTENSION**

Carry out a statistical investigation of their own, including designing an appropriate means of gathering the data  
 Some guidance needs to be given to stop students from choosing limited investigations, eg favourite football team  
 Get data from holiday brochures to compare resorts for temp, rainfall and type of visitor  
 Carry out a statistical investigation of their own including, designing an appropriate means of gathering the data  
 Investigation into other sampling schemes, such as cluster and quota sampling

## NOTES

Students may need reminding about the correct use of tallies

Emphasise the differences between primary and secondary data

Discuss sample size and mention that a census is the whole population

In the UK it is every year that ends in a '1' (2011 is the next census)

If students are collecting data as a group, they should all use the same procedure

Emphasise that continuous data is data that is measured, eg temperature

Mayfield High data from coursework task can be used to collect samples and can be used to make comparisons in following sections.

Use year group data, eg Mayfield High data to introduce stratified sampling techniques

Use investigations to link with future statistics modules

GCSE Tier: Higher

Contents: Displaying data

SP g	Produce charts and diagrams for various data types
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP j	Present findings from databases, tables and charts
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l	Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts

**PRIOR KNOWLEDGE:**

An understanding of the different types of data: continuous; discrete;  
 Experience of inequality notation  
 Ability to multiply a number by a fraction  
 Use a protractor to measure and draw angles

**OBJECTIVES**

By the end of the module the student should be able to:

- Produce: Pie charts, frequency polygons, histograms with equal class intervals and frequency diagrams for grouped discrete data, scatter graphs, line graphs, frequency polygons for grouped data, grouped frequency tables for continuous data (18.1, 18.2, 18.4 - 18.9, 24.2)
- Interpret: composite bar charts, comparative and dual bar charts, pie charts, scatter graphs, frequency polygons and histograms (18.1, 18.2, 18.4 - 18.7, 24.2 - 24.5)
- Recognise simple patterns, characteristics and relationships in line graphs and frequency polygons (18.6, 24.1)
- Find the median from a histogram or any other information from a histogram, such as the number of people in a given interval (18.5, 18.7)
- From line graphs: read off frequency values, calculate total population, find greatest and least values (18.6, 18.8, 18.9, 24.1)
- From pie charts: find the total frequency and find the frequency represented by each sector (18.2)
- From histograms: complete a grouped frequency table and understand and define frequency density (18.5 - 18.7)
- Present findings from databases, tables and charts (Chapter 18)
- Look at data to find patterns and exceptions, explain an isolated point on a scatter graph (Chapter 18, 24.3)
- Draw lines of best fit by eye, understanding what these represent. Use a line of best fit, or otherwise, to predict values of one variable given values of the other variable (24.4, 24.5)
- Distinguish between positive, negative and zero correlation using lines of best fit (24.3)
- Understand that correlation does not imply causality (24.3)
- Appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship'

**DIFFERENTIATION & EXTENSION**

Carry out a statistical investigation of their own and use an appropriate means of displaying the results  
 Use a spreadsheet/ICT to draw different types of graphs

**NOTES**

Collect examples of charts and graphs in the media which have been misused, and discuss the implications  
 Clearly label all axes on graphs and use a ruler to draw straight lines  
 Many students enjoy drawing statistical graphs for classroom displays. Include the Functional Elements in this topic with regard to holiday data, energy charts etc  
 Stem and leaf diagrams must have a key and show how to find the median and mode from a stem and leaf diagram.  
 Angles for pie charts should be accurate to within 2°. Ask students to check each others' charts  
 Make comparisons between previously collected data, eg Mayfield boys vs girls or Yr 7 vs Yr 8  
 Encourage students to work in groups and present their charts – display work in classroom/corridors  
 Use Excel Graph wizard  
 Data collected in Module 6 can then be displayed as part of the work in this module

**Module** 8

**Time: 4 – 6 hours**

**GCSE Tier: Higher**

**Contents: Constructions and Loci**

GM v Use straight edge and a pair of compasses to carry out constructions

GM w Construct loc

**PRIOR KNOWLEDGE:**

An ability to use a pair of compasses

The special names of triangles (and angles)

Understanding of the terms perpendicular, parallel and arc

**OBJECTIVES**

By the end of the module students should be able to:

- Use straight edge and a pair of compasses to do standard constructions (10.1, 10.2)
- Construct triangles inc an equilateral triangle (10.5)
- Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not
- Construct the perpendicular bisector of a given line (10.1)
- Construct the perpendicular from a point to a line (10.1)
- Construct the perpendicular from a point on a line. (10.1)
- Construct the bisector of a given angle (10.2)
- Construct angles of  $60^\circ$ ,  $90^\circ$ ,  $30^\circ$ ,  $45^\circ$
- Draw parallel lines
- Draw circles and arcs to a given radius
- Construct a regular hexagon inside a circle
- Construct diagrams of everyday 2-D situations involving rectangles, triangles, perpendicular and parallel lines.
- Draw and construct diagrams from given information
- Construct: -a region bounded by a circle and an intersecting line
  - given distance from a point and a given distance from a line (10.3)
  - equal distances from 2 points or 2 line segments (10.3)
  - regions which may be defined by ‘nearer to’ or ‘greater than’ (10.4)
- Find and describe regions satisfying a combination of loci (10.4)

**DIFFERENTIATION & EXTENSION**

Solve loci problems that require a combination of loci

Relate to real life examples including horses tethered in fields and mobile phone masts

**NOTES**

All working should be presented clearly, and accurately

A sturdy pair of compasses are essential

Construction lines should not be erased as they carry method marks

Could use construction to link to similarity and congruence

GCSE Tier: Higher

Contents: Types of number (Factors, Multiples, Primes and Powers and Roots)

N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Least Common Multiple, prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers
N g	Interpret, order and calculate with numbers written in standard form
N v	Use a calculator effectively and efficiently

**PRIOR KNOWLEDGE:**

Number complements to 10 and multiplication and division facts  
 Use a number line to show how numbers relate to each other  
 Recognise basic number patterns  
 Experience of classifying integers

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify factors, multiples and prime numbers from a list of numbers (1.2)
- Find the prime factor decomposition of positive integers (1.2)
- Find the common factors and common multiples of two or three numbers (1.2)
- Find the Highest Common Factor (HCF) and the Least Common Multiple (LCM) of two or three numbers (1.2)
- Recall integer squares from  $2 \times 2$  to  $15 \times 15$  and the corresponding square roots (1.4)
- Recall the cubes of 2, 3, 4, 5 and 10 (1.4)
- Use index notation for squares and cubes (1.4)
- Use index notation for integer powers of 10 (25.5)
- Use standard form, expressed in conventional notation (25.2)
- Be able to write very large and very small numbers presented in a context in standard form (25.2)
- Convert between ordinary and standard form representations (25.2)
- Calculate with standard form (25.2)
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, and powers of a power (1.7)

**DIFFERENTIATION & EXTENSION**

Calculator exercise to check factors of larger numbers  
 Further work on indices to include negative and/or fractional indices  
 Use prime factors to find LCM and square roots  
 Plenty of investigative work for squares like 'half time' scores  
 Use a number square to find primes (sieve of Eratosthenes)  
 Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

**NOTES**

All of the work in this unit is easily reinforced by starters and plenaries  
 Calculators are used only when appropriate  
 Encourage student to learn square, cube, prime and common roots for the non-calculator examination

**Module** 10

**Time: 3 – 5 hours**

**GCSE Tier: Higher**

**Contents: Patterns and sequences**

A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence

A j Use linear expressions to describe the  $n^{\text{th}}$  term of an arithmetic sequence

**PRIOR KNOWLEDGE:**

Know about odd and even numbers

Recognise simple number patterns eg 1, 3, 5, ...

Writing simple rules algebraically

Raise numbers to positive whole number powers

**OBJECTIVES**

By the end of the module the student should be able to:

- Recognise sequences of odd and even numbers (2.6)
- Generate simple sequences of numbers, squared integers and sequences derived from diagrams (2.6)
- Describe the term-to-term definition of a sequence in words (2.6)
- Identify which terms cannot be in a sequence (2.6, 2.7)
- Generate specific terms in a sequence using the position-to-term and term-to-term rules (2.6, 2.7)
- Find the  $n^{\text{th}}$  term of an arithmetic sequence (2.7)
- Use the  $n^{\text{th}}$  term of an arithmetic sequence (2.7)

**DIFFERENTIATION & EXTENSION**

Emphasise good use of notation, eg  $3n$  means  $3 \times n$

When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the  $n^{\text{th}}$  term

Match-stick problems

Sequences and nth term formula for triangle numbers, Fibonacci numbers etc

Prove a sequence cannot have odd numbers for all values of  $n$

Extend to quadratic sequences whose  $n^{\text{th}}$  term is  $an^2 + bn + c$

**NOTES**

Emphasis on good use of notation  $3n$  means  $3 \times n$

When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the  $n^{\text{th}}$  term

Module 11

Time: 3 – 5 hours

GCSE Tier: Higher

Contents: 2-D and 3-D shapes

GM k Use 2-D representations of 3-D shapes

**PRIOR KNOWLEDGE:**

Construction and loci

**OBJECTIVES**

By the end of the module the student should be able to:

- Use 2-D representations of 3-D shapes (12.6 - 12.9)
- Use isometric grids (12.6)
- Draw nets and show how they fold to make a 3-D solid (12.6)
- Understand and draw front and side elevations and plans of shapes made from simple solids (12.7)
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3-D solid (12.7)

**DIFFERENTIATION & EXTENSION**

Make solids using equipment such as clixi or multi-link with different coloured cubes.

Draw shapes made from multi-link on isometric paper

Construct combinations of 2-D shapes to make nets

Build shapes from cubes that are represented in 2-D

An excellent topic for wall display.

Extend to Planes of Symmetry for 3-D solids

Discover Euler’s Formula for solids

Investigate how many small boxes can be packed into a larger box, as a Functional-type example

This result is known as **Euler’s formula**. An illustration of the formula on some below.

Name	Image	Vertices <i>V</i>	Edges <i>E</i>	Faces <i>F</i>	Euler characteristic: $V - E + F$
Tetrahedron		4	6	4	2
Hexahedron or cube		8	12	6	2
Octahedron		6	12	8	2
Dodecahedron		20	30	12	2
Icosahedron		12	30	20	2

**NOTES**

All working should be presented clearly, and accurately

A sturdy pair of compasses are essential

Accurate drawing skills need to be reinforced

Some students find visualising 3-D objects difficult- simple models will assist

**Module** 12

**Time: 6 – 8 hours**

**GCSE Tier: Higher**

**Contents: Perimeter and area**

GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM z	Find circumferences and areas of circles
N r	Use $\pi$ in an exact calculation
GM bb	Solve mensuration problems involving more complex shapes and solids
GM p	Convert measurements from one unit to another

**PRIOR KNOWLEDGE:**

Names of triangles, quadrilaterals and polygons  
Concept of perimeter and area  
Units of measurement  
Substitute numbers into formulae  
Ability to give answers to a degree of accuracy

**OBJECTIVES**

By the end of the module the student should be able to:

- Measure sides of a rectangle to work out perimeter or area (12.1)
- Calculate perimeters of shapes made from triangles and rectangles (12.1)
- Find the perimeter of rectangles and triangles (12.1)
- Calculate areas of shapes made from triangles and rectangles (12.3)
- Recall and use the formulae for the area of a triangle, rectangle and a parallelogram (12.2)
- Find the area of a trapezium and a parallelogram (12.2)
- Calculate perimeter and area of compound shapes made from triangles, rectangles and other shapes
- Find the surface area of simple shapes (prisms) using the formulae for triangles and rectangles, and other shapes (23.8)
- Find circumferences of circles and areas enclosed by circles (12.4, 12.5)
- Recall and use the formulae for the circumference of a circle and the area enclosed by a circle (12.4, 12.5)
- Use  $\pi \approx 3.142$  or use the  $\pi$  button on a calculator (12.4, 12.5)
- Give an exact answer to a question involving the area of a circle (23.2)
- Find the perimeters and areas of semicircles and quarter circles (23.1)
- Calculate the lengths of arcs and the areas of sectors of circles (23.1)
- Answers in terms of  $\pi$  may be required (23.2)
- Find the surface area of a cylinder (23.9)
- Find the area of a segment of a circle given the radius and length of the chord
- Convert between units of area (23.3)

**DIFFERENTIATION & EXTENSION**

Calculate areas and volumes using formulae  
Using compound shape methods to investigate areas of other standard shapes such as parallelograms, trapeziums and kites  
Emphasise the functional elements here with carpets for rooms, tiles for walls, turf for gardens as well as wall paper and skirting board problems  
Further problems involving combinations of shapes  
Practical activities, eg using estimation and accurate measuring to calculate perimeters and areas of classroom/corridor floors

**NOTES**

Discuss the correct use of language and units  
Ensure that students can distinguish between perimeter, area and volume  
Practical experience is essential to clarify these concepts.  
There are many Functional Elements questions which can be applied to this topic area, eg floor tiles, optimization type questions, which pack of tiles give the best value?

**Module 13**

**Time: 7 – 9 hours**

**GCSE Tier: Higher**

**Contents: Fractions, decimals and percentages**

N j	Use decimal notation and recognise that each terminating decimal is a fraction
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N l	Understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions
N m	Use percentage, repeated proportional change
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

- Four operations of number
- The concepts of a fraction and a decimal
- Awareness that percentages are used in everyday life

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths (14.1)
- Convert between fractions decimals and percentages (14.1)
- Convert between recurring decimals and exact fractions as well as understanding the proof (4.4)
- Write one number as a percentage of another number (14.1)
- Calculate the percentage of a given amount (14.1, 14.2)
- Find a percentage increase/decrease of an amount (14.3)
- Find a reverse percentage, eg find the original cost of an item given the cost after a 10% deduction (14.5)
- Use a multiplier to increase by a given percent over a given time , eg  $1.1^8 \times 64$  increases 64 by 10% over 8 years (14.4)
- Calculate simple and compound interest (14.4)

**DIFFERENTIATION & EXTENSION**

- Find fractional percentages of amounts, without using a calculator, eg 0.825%
- Combine multipliers to simplify a series of percentage changes
- Percentages which convert to recurring decimals (eg  $33\frac{1}{3}\%$ ), and situations which lead to percentages of more than 100%
- Problems which lead to the necessity of rounding to the nearest penny (eg real-life contexts)
- Comparisons between simple and compound interest calculations

**NOTES**

- Emphasise the Functional Elements in this topic, use real-world problems involving fractions, decimals and percentages
- Amounts of money should always be rounded to the nearest penny where necessary, except where such rounding is premature, eg in successive calculations like in compound interest
- In preparation for this unit, students should be reminded of basic percentages and recognise their fraction and decimal equivalents
- Link with probability calculations using AND and OR Laws
- Use fractions for calculations involving compound units

**Module 14**

**Time: 6 – 8 hours**

**GCSE Tier: Higher**

**Contents: Formulae and solution of linear equations**

- A f Derive a formula, substitute numbers into a formula and change the subject of a formula  
A d Set up and solve simple equations  
A g Solve linear inequalities in one variable, and represent the solution set on a number line

**PRIOR KNOWLEDGE:**

- Experience of finding missing numbers in calculations
- The idea that some operations are ‘opposite’ to each other
- An understanding of balancing
- Experience of using letters to represent quantities
- Understand and recall BIDMAS

**OBJECTIVES**

By the end of the module the student should be able to:

- Derive a formula (19.6)
- Use formulae from mathematics and other subjects (19.5)
- Substitute numbers into a formula (19.5, 2.3)
- Substitute positive and negative numbers into expressions such as  $3x^2 + 4$  and  $2x^3$  (19.5)
- Set up linear equations from word problems (13.5, 19.6)
- Solve simple linear equations (13.1, 13.5)
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation (13.1, 13.1)
- Solve linear equations that include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution (13.2)
- Solve linear equations in one unknown, with integer or fractional coefficients (13.1 - 13.4)
- Solve simple linear inequalities in one variable, and represent the solution set on a number line (19.1, 19.2)
- Use the correct notation to show inclusive and exclusive inequalities (19.1, 19.2)
- Change the subject of a formula including cases where the subject is on both sides of the original formula, or where a power of the subject appears (19.7, 19.8)

**DIFFERENTIATION & EXTENSION**

- Solve second order linear equations
- Use negative numbers in formulae involving indices
- Use investigations to lead to generalisations
- Apply changing the subject to  $y = mx + c$
- Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)
- Solve equations where manipulation of fractions (including the negative fractions) is required

**NOTES**

- Emphasise good use of notation  $3ab$  means  $3 \times a \times b$
- Students need to be clear on the meanings of the words expression, equation, formula and identity
- Students need to realise that not all linear equations can easily be solved by either observation or trial and improvement, and hence the use of a formal method is important
- Students can leave their answers in fractional form where appropriate
- Students should use the correct notation when giving graphical solutions to inequalities, eg a dotted boundary line for  $<$  or  $>$

GCSE Tier: Higher

Contents: Linear graphs

- A l Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
- A m Understand that the form  $y = mx + c$  represents a straight line and that  $m$  is the gradient of the line and  $c$  is the value of the  $y$  - intercept
- A n Understand the gradients of parallel lines
- A g Solve linear inequalities in two variables, and represent the solution set on a coordinate grid
- A r Construct linear functions from real-life problems and plot their corresponding graphs

**PRIOR KNOWLEDGE**

Being able to:

- Substitute positive and negative numbers into algebraic expressions
- Plot coordinates in the first quadrant
- Rearrange to change the subject of a formula

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw, label and scale axes (Chapter 15)
- Recognise that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane (15.4)
- Draw straight line graphs for real-life situations (15.6)
  - ready reckoner graphs
  - conversion graphs
  - fuel bills, eg gas and electric
  - fixed charge (standing charge) and cost per unit
- Plot and draw graphs of straight lines with equations of the form  $y = mx + c$  (15.4, 15.5)
- Find the gradient of a straight line from a graph (15.3)
- Analyse problems and use gradients to interpret how one variable changes in relation to another (15.3)
- Find the gradient of lines given by equations of the form  $y = mx + c$  (15.3, 15.4)
- Understand that the form  $y = mx + c$  represents a straight line (15.4)
- Find the gradient of a straight line from its equation (15.4)
- Explore the gradients of parallel lines and lines perpendicular to each other (15.5)
- Write down the equation of a line parallel or perpendicular to a given line (15.5)
- Use the fact that when  $y = mx + c$  is the equation of a straight line then the gradient of a line parallel to it will have a gradient of  $m$  and a line perpendicular to this line will have a gradient of  $-\frac{1}{m}$  (15.5)
- Interpret and analyse a straight line graph and generate equations of lines parallel and perpendicular to the given line (15.5)
- Show the solution set of several inequalities in two variables on a graph (19.4)

**DIFFERENTIATION & EXTENSION**

- Find the equation of the line through two given points
- Find the equation of the perpendicular bisector of the line segment joining two given points
- Use Functional Elements in terms of mobile phone bills
- Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
- Use a graphical calculator or graphical ICT package to draw straight-line graphs
- Link to scatter graphs and correlation
- Cover horizontal and vertical lines ( $x = c$  and  $y = c$ ), as students often forget these

## NOTES

Careful annotation should be encouraged. Label the coordinate axes and write the equation of the line

Recognise linear graphs and hence when data may be incorrect

Link to graphs and relationships in other subject areas, i.e. science, geography etc

Interpret straight line graphs for Functional Elements problems

- Ready reckoner graphs
- Conversion graphs
- Fuel bills and mobile phone tariffs
- Fixed charge (standing charge) and cost per unit

Link conversion graphs to converting metric and imperial units

A-Level C1 text books can be a good source of extension questions on this topic

Students should use correct notation to show inequalities in algebra and graphs

Module 16

Time: 3 – 5 hours

GCSE Tier: Higher

Contents: Simultaneous equations

A d Set up and solve simple equations including simultaneous equations in two unknowns

**PRIOR KNOWLEDGE:**

Introduction to algebra

Linear functions

**OBJECTIVES**

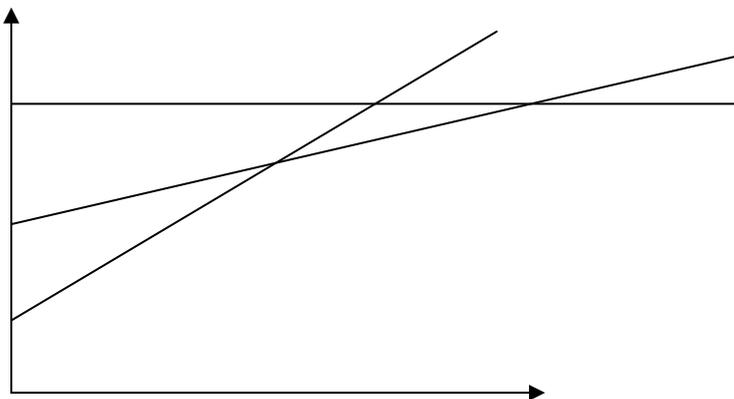
By the end of the module the student should be able to:

- Find the exact solutions of two simultaneous equations in two unknowns (22.1)
- Use elimination or substitution to solve simultaneous equations (22.1)
- Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection. Consider the real life applications, eg mobile phone bills (22.3)
- Set up and solve a pair of simultaneous equations in two variables (22.2)

**DIFFERENTIATION & EXTENSION**

Inaccurate graphs could lead to incorrect solutions

Clear presentation of workings is essential



Use open ended questions that test student awareness of what intersections mean for mobile phone bills

Solve two simultaneous equations with fractional coefficients

Solve two simultaneous equations with second order terms, eg equations in  $x$  and  $y^2$

**NOTES**

Build up the algebraic techniques slowly

Link the graphical solutions with linear graphs and changing the subject

Inaccurate graphs could lead to incorrect solutions, encourage substitution of answers to check they are correct

Clear presentation of working is essential

GCSE Tier: Higher

Contents: Probability

SP m	Understand and use the vocabulary of probability and probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP q	Know when to add or multiply two probabilities: when $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$ , whereas when $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$
SP r	Use tree diagrams to represent outcomes of compound events, recognising when events are independent
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

**PRIOR KNOWLEDGE:**

Understand that a probability is a number between 0 and 1  
 Know how to add, and multiplying fractions and decimals  
 Experience of expressing one number as a fraction of another number  
 Recognise the language of statistics, eg words such as likely, certain, impossible

**OBJECTIVES**

By the end of the module the student should be able to:

- Write probabilities using fractions, percentages or decimals (28.1)
- Understand and use estimates or measures of probability (28.1)
- Use theoretical models to include outcomes using dice, spinners, coins etc (28.1, 28.3)
- Understand the probability of successive events, such as several throws of a single dice (28.1)
- List all outcomes for single events, and for two successive events, systematically (28.1)
- Use and draw sample space diagrams (28.1)
- Add simple probabilities, eg from sample space diagrams (28.2)
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 (28.2)
- Use  $1 - p$  as the probability of an event not occurring where  $p$  is the probability of the event occurring (28.2)
- Find a missing probability from a list or table (28.2)
- Understand conditional probabilities (28.7)
- Understand selection with or without replacement (28.7)
- Draw a probability tree diagram based on given information (28.6)
- Use a tree diagram to calculate conditional probability (28.7)
- Compare experimental data and theoretical probabilities (28.3, 28.4)
- Compare relative frequencies from samples of different sizes (28.3, 28.4)

**DIFFERENTIATION & EXTENSION**

An opportunity for practical examples, eg  $P(\text{pin up})$  for a drawing pin, the ‘horse’ race, the national lottery  
 Show that each cluster of branches adds up to 1

Explain that if two objects are chosen, then this is the same as one event followed by another event without replacement

Show that it is often easier to solve a problem involving multiple outcomes, by considering the *opposite* event and subtracting from 1, eg ‘at least’ two reds, ‘at least’ two beads of a different colour etc)

Experiments with dice and spinners

Show sample space for outcomes of throwing 2 dice.

Stress that there are 36 outcomes (they will initially guess it’s 12 outcomes for 2 dice)

Binomial probabilities (H or T)

Do a question ‘with’ and then repeat it ‘without’ replacement. Good idea to show the contents of the bag and physically remove the object to illustrate the change of probability fraction for the second selection

**NOTES**

Students should express probabilities as fractions, percentages or decimals

Fractions needed not be cancelled to their lowest terms. This makes it easier to calculate tree diagram probabilities, eg easier to add like denominators

Show that each cluster of branches adds up to 1. Also ALL the outcomes add up to 1 too.

**Module 18**

**Time: 6 – 8 hours**

**GCSE Tier: Higher**

**Contents: Ratio and scale**

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N t	Divide a quantity in a given ratio
GM m	Use and interpret maps and scale drawings
N n	Understand and use direct and indirect proportion
A u	Direct and indirect proportion (algebraic)

**PRIOR KNOWLEDGE:**

Fractions

**OBJECTIVES**

By the end of the module the student should be able to:

- Use ratios (16.1)
- Write ratios in their simplest form (16.1)
- Divide a quantity in a given ratio (16.3)
- Solve a ratio problem in a context (16.2)
- Use and interpret maps and scale drawings (10.6)
- Read and construct scale drawings drawing lines and shapes to scale (10.6)
- Estimate lengths using a scale diagram (10.6)
- Solve word problems about ratio and Proportion (16.2 - 16.5)
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion (16.4, 16.5, 27.1)
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations (16.4, 16.5, 27.1 - 27.5)

**DIFFERENTIATION & EXTENSION**

Harder problems involving multi-stage calculations

Relate ratios to real-life situations, eg investigate the proportions of the different metals in alloys (24 karat gold), Best buy type questions as well as recipes, currency calculations using currency exchange rates

Relate ratios to Functional Elements situations, eg investigate the proportions of the different metals in alloys and the new amounts of ingredients for a recipe for different numbers of guests

Harder problems involving multi-stage calculations

**NOTES**

Students often find ratios with three parts difficult

Link ratios given in different units to metric and imperial units

GCSE Tier: Higher

Contents: Averages and Range

SP h	Calculate median, mean, range, quartiles and interquartile range, mode and modal class
SP g	Produce charts and diagrams for various data types
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP l	Compare distributions and make inferences
SP u	Use calculators efficiently and effectively, including statistical functions

**PRIOR KNOWLEDGE:**

Knowledge of finding the mean for small data sets  
 Ability to find the midpoint of two numbers

**OBJECTIVES**

By the end of the module the student should be able to:

- Calculate mean, mode, median and range for small data sets (11.1, 11.2, 11.7)
- Recognise the advantages and disadvantages between measures of average (11.3)
- Produce ordered stem and leaf diagrams and use them to find the range and averages (18.3)
- Calculate averages and range from frequency tables (Use  $\Sigma x$  and  $\Sigma fx$ ) (11.4)
- Estimate the mean for large data sets with grouped data (and understand that it is an estimate) (11.6)
- Draw cumulative frequency tables and graphs (18.8)
- Use cumulative frequency graphs to find median, quartiles and interquartile range (18.9, 11.7)
- Draw box plots from a cumulative frequency graph (18.10)
- Compare the measures of spread between a pair of box plots/cumulative frequency graphs (18.8 - 18.10)
- Interpret box plots to find median, quartiles, range and interquartile range (18.10, 11.7)
- Find the median from a histogram (18.5, 11.5)
- Compare distributions and make inferences, using the shapes of distributions and measures of average and spread, including median and quartiles (11.7, 18.9, 18.10)

**DIFFERENTIATION & EXTENSION**

Use statistical functions on calculators and spreadsheets

Use statistical software to calculate the mean for grouped data sets

Estimate the mean for data sets with ill defined class boundaries

Investigate the affect of combining class intervals on estimating the mean for grouped data sets

Students should understand that finding an *estimate for the mean* of grouped data is not a guess

Opportunity to remind them of Module 6

Pose the question: ‘Investigate if the average number of children per family is 2.4.’, ‘Are the families represented in your class representative of the whole population?’

Discuss occasions when one average is more appropriate, and the limitations of each average

Possibly mention standard deviation (not on course, but good for further comparison of data sets with similar means)

**NOTES**

Collect data from class – children per family etc. Extend to different classes, Year groups or secondary data from the internet. (Previous coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*)

Compare distributions and make inferences, using the shapes of distributions and measures of average and spread, eg ‘boys are taller on average but there is a much greater spread in heights’ (Use data collected from previous investigations or Mayfield High data)

Students tend to select modal class but identify it by the frequency rather than the class itself

Explain that the median of grouped data is not necessarily from the middle class interval



**Module 21**

**Time: 3 – 5 hours**

**GCSE Tier: Higher**

**Contents: Trial and improvement**

- A h Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- N u Approximate to specified or appropriate degrees of accuracy including a number of decimal places and significant figures
- N v Use calculators effectively and efficiently

**PRIOR KNOWLEDGE:**

Substituting numbers into algebraic expressions  
Dealing with decimals on a calculator  
Ordering decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve cubic functions by successive substitution of values of  $x$  (21.5)
- Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them (21.5)
- Understand the connections between changes of sign and location of roots

**DIFFERENTIATION & EXTENSION**

Solve functions of the form  $\frac{1}{x} = x^2 - 5$  (link with changing the subject)

Students should be encouraged to use their calculators efficiently – by using the ‘replay’ function

Students should write down all the digits on their calculator display

Look at ‘practical examples’. A room is 2 m longer than it is wide. If its area is 30 m<sup>2</sup> what is its perimeter?

**NOTES**

Students should be encouraged to use their calculators efficiently – by using the "replay" or ANS/EXE functions

The square/cube function on a calculator may not be the same for different makes

Take care when entering negative values to be squared (always use brackets)

Students should write down all the digits on their calculator display and only round the final answer declared to the degree of accuracy

GCSE Tier: Higher

Contents: Surface area and volume

GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM z	Find the surface area of a cylinder
GM bb	Solve mensuration problems involving more complex shapes and solids
GM p	Convert measures from one unit to another
N r	Use $\pi$ in exact calculations

**PRIOR KNOWLEDGE:**

- Concept of volume
- Knowledge of area module
- Ability to give answers to a degree of accuracy
- Experience of changing the subject of a formula

**OBJECTIVES**

By the end of the module the student should be able to:

- Know and use formulae to calculate the surface areas and volumes of cuboids and right-prisms (12.8, 12.9)
- Solve a range of problems involving surface area and volume, eg given the volume and length of a cylinder find the radius (Chapter 23, 12.8 - 12.10)
- Find the volume of a cylinder and surface area of a cylinder (12.10, 23.9)
- Convert between volume measures, including cubic centimetres and cubic metres (23.7)
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones (23.6)
- Find the surface area and volumes of compound solids constructed from; cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinder, eg solids in everyday use (23.4 - 23.6, 23.8 - 23.9)

**DIFFERENTIATION & EXTENSION**

- Additional work using algebraic expressions
- Find surface area and volume of a sphere or cone (using standard formulae)
- Convert between less familiar units, eg  $\text{cm}^3$  to  $\text{mm}^3$ ,  $\text{cm}^3$  to litres
- Answers in terms of  $\pi$  may be required
- Look at functional type questions, eg fitting boxes in crates
- Look at in conjunction with Module 23 and density/volume/mass questions
- Find the volume of a cylinder given its surface area, leaving the answer in terms of  $l$
- Find the volume of a right hexagonal side  $x$  and height  $h$  (researching the method for finding the volume of any cone)

**NOTES**

- 'Now! I Know Pi' is a good way to learn the approx value (The number of letters of each word and the ! is the decimal point)
- Also 'Cherry Pie Delicious' is  $C = \pi D$  and 'Apple Pies are too' is  $A = \pi r^2$
- Answers in terms of  $\pi$  may be required or final answers rounded to the required degree of accuracy
- Need to constantly revise the expressions for area/volume of shapes
- Students should be aware of which formulae are on the relevant page on the exam paper and which they need to learn

GCSE Tier: Higher

Contents: Compound measures

GM o	Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM q	Make sensible estimates of a range of measures
GM s	Understand and use compound measures
A r	Construct distance time graphs
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
N s	Calculate upper and lower bounds

**PRIOR KNOWLEDGE:**

- Knowledge of metric units, eg 1 m = 100 cm etc
- Know that 1 hour = 60 mins, 1 min = 60 seconds
- Experience of multiply by powers of 10, e.g.  $100 \times 100 = 10\,000$

**OBJECTIVES**

By the end of the module the student should be able to:

- Convert between units of measure in the same system. (NB: Conversion between imperial units will be given, metric equivalents should be known) (7.3)
- Know rough metric equivalents of pounds, feet, miles, pints and gallons: Metric /Imperial 1 kg= 2.2 pounds, 1 litre=1.75 pints, 4.5l= 1 gallon, 8 km = 5 miles, 30 cm = 1 foot (7.3)
- Convert between imperial and metric measures (7.3)
- Use the relationship between distance, speed and time to solve problems (7.4, 7.5)
- Convert between metric units of speed eg km/h to m/s (7.5)
- Construct and interpret distance time graphs (15.6)
- Know that density is found by mass  $\div$  volume (7.6)
- Use the relationship between density, mass and volume to solve problems, eg find the mass of an object with a given volume and density (7.6)
- Convert between metric units of density eg kg/m to g/cm (7.6)
- Calculate speed when, eg fractions of an hour must be entered as fractions or as decimals (7.5)
- Calculate the upper and lower bounds of calculations, particularly when working with measurements (4.9, 4.10)
- Find the upper and lower bounds of calculations involving perimeter, areas and volumes of 2-D and 3-D shapes (4.10)
- Find the upper and lower bounds in real life situations using measurements given to appropriate degrees of accuracy (4.10)
- Give the final answer to an appropriate degree of accuracy following an analysis of the upper and lower bounds of a calculation (4.10)

**DIFFERENTIATION & EXTENSION**

- Perform calculations on a calculator by using standard form
- Convert imperial units to metric units, eg mph into km/h
- Use a distance, speed and time (or mass, density and volume) triangle to help students see the relationship between the variables
- Help students to recognise the problem they are trying to solve by the unit measurement given, eg km/h is a unit of speed as it is a distance divided by a time
- Mention other units (not on course) like hectares

**NOTES**

- Use a formula triangle to help students see the relationship between the variables for density
- Borrow a set of electronic scales and a Eureka Can from Physics for a practical density lesson.
- Look up densities of different elements from the net.
- Link converting area & volume units to similar shapes (Module 25)
- Draw a large grid made up of 100 by 100 cm squares to show what 1 square metre looks like

GCSE Tier: Higher

Contents: Transformations

GM e Recognise reflection and rotation symmetry of 2-D shapes

GM 1 Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, fractional or negative scale factor and distinguish properties that are preserved under particular transformations

**PRIOR KNOWLEDGE:**

Recognition of basic shapes

An understanding of the concept of rotation, reflection and enlargement

Coordinates in four quadrants

Linear equations parallel to the coordinate axes

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand translation as a combination of a horizontal and vertical shift including signs for directions (17.1, 17.2)
- Translate a given shape by the vector (17.2)
- Understand rotation as a (anti clockwise) turn about a given origin (17.1, 17.4)
- Reflect shapes in a given mirror line; parallel to the coordinate axes and then  $y = x$  or  $y = -x$  (17.3)
- Enlarge shapes by a given scale factor from a given point; using positive and negative scale factors greater and less than one (17.5, 17.6)
- Find the centre of enlargement (17.6)
- Understand that shapes produced by translation, rotation and reflection are congruent to the image Describe and transform 2-D shapes using single rotations (17.1 – 17.4)
- Understand that rotations are specified by a centre and an (anticlockwise) angle; questions will specify clockwise or anticlockwise when required (17.4)
- Find the centre of rotation (17.4)
- Rotate a shape about the origin, or any other point (17.4)
- Describe and transform 2-D shapes using combined rotations, reflections, translations, or enlargements (17.7)
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations (17.1 - 17.4)
- Distinguish properties that are preserved under particular transformations (Chapter 17)
- Recognise that enlargements preserve angle but not length, linking to SIMILARITY (17.5, 17.6)
- Describe a transformation (Chapter 17)

**DIFFERENTIATION & EXTENSION**

The tasks set can be extended to include combinations of transformations

Diagrams should be drawn carefully

**NOTES**

Emphasise that students describe the given transformation fully

Diagrams should be drawn carefully

The use of tracing paper is allowed in the examination (although students should not have to rely on the use of tracing paper to solve problems)

Module 25

Time: 4 – 6 hours

GCSE Tier: Higher

Contents: Similarity and Congruence

GM f Understand congruence and similarity  
GM n Understand and use the effect of enlargement for perimeter, area and volume of shapes and solids  
N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

**PRIOR KNOWLEDGE:**

Ratio  
Proportion  
Area and Volume

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and a pair of compasses constructions (15.1)
- Understand similarity of triangles and of other plane figures, and use this to make geometric inferences (15.2)
- Formal geometric proof of similarity of two given triangles (15.2, 15.3)
- Recognise that all corresponding angles in similar figures are equal in size when the lengths of sides are not (15.2)
- Understand the effect of enlargement for perimeter, area and volume of shapes and solids (26.1-26.3)
- Understand that enlargement does not have the same effect on area and volume (26.1-26.3)
- Use simple examples of the relationship between enlargement and areas and volumes of simple shapes and solids (26.1-26.3)
- Use the effect of enlargement on areas and volumes of shapes and solids (26.1-26.3)
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids (26.1-26.3)
- Understand that the inverse operation of raising a positive number to a power  $n$  is raising the result of this operation to the power  $\frac{1}{n}$  (20.1)

**DIFFERENTIATION & EXTENSION**

This could be introduced practically or by investigation simple shapes such as squares, rectangles, circles (reminder of formula), cuboids, cylinders etc  
Solve loci problems that require a combination of loci  
Construct combinations of 2-D shapes to make nets  
Link with tessellations and enlargements  
Link with similar areas and volumes  
Harder problems in congruence  
Relate this unit to circle theorems

**NOTES**

All working should be presented clearly, and accurately  
A sturdy pair of compasses are essential  
Construction lines should not be erased as they carry method marks

GCSE Tier: Higher

Contents: Quadratic functions, equations and graphs

- A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
- A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
- A r Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
- A e Solve quadratic equations
- A o Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

**PRIOR KNOWLEDGE:**

Graphs and algebra

**OBJECTIVES**

By the end of the module the student should be able to:

- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions (21.1)
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function (21.1)
- Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions (22.9)
- Solve simple quadratic equations by factorisation and completing the square (22.4, 22.5, 22.7, 22.8, 32.4)
- Solve simple quadratic equations by using the quadratic formula (22.6 - 22.8, 32.4)
- Select and apply algebraic and graphical techniques to solve simultaneous equations where one is linear and one quadratic (22.9, 2.10)
- Solve equations involving algebraic fractions which lead to quadratic equations
- Solve quadratic equations by completing the square
- Derive the quadratic equation by completing the square

**DIFFERENTIATION & EXTENSION**

Derive the quadratic equation by completing the square

Use graphical calculators or ICT graph package where appropriate

Show how the value of ' $b^2 - 4ac$ ' can be useful in determining if the quadratic factorises or not (i.e. square number)

Extend to discriminant's properties and roots

**NOTES**

Lots of practical type examples, eg help projectiles

Some students may need additional help with factorising

Students should be reminded that factorisation should be tried before the formula is used

In problem-solving, one of the solutions to a quadratic may not be appropriate

There may be a need to remove the HCF (numerical) of a trinomial before factorising to make the factorisation easier

In problem-solving, one of the solutions to a quadratic may not be appropriate. eg negative length

GCSE Tier: Higher

Contents: Index notation and surds

N e	Use index notation for squares, cubes and powers of 10
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N f	Use index laws for multiplication and division of integer, fractional and negative powers
N v	Use calculators effectively and efficiently
N r	Calculate with surds

**PRIOR KNOWLEDGE:**

Knowledge of squares, square roots, cubes and cube roots  
Fractions and algebra 1

**OBJECTIVES**

By the end of the module the student should be able to:

- Use index laws to simplify and calculate numerical expressions involving powers, eg  $(2^3 \times 2^5) \div 2^4$ ,  $4^0$ ,  $8^{-2/3}$  (25.1, 25.3)
- Know that, eg  $x^3 = 64 \Rightarrow x = 8^{2/3}$  (25.3)
- Rationalise the denominator, eg  $\frac{1}{\sqrt{3}-1} = \left(\frac{\sqrt{3}+1}{2}\right)$ , and, eg write  $(\sqrt{18}+10) \div \sqrt{2}$  in the form  $p + q\sqrt{2}$  (25.4)
- Use calculators to explore exponential growth and decay (21.4)

**DIFFERENTIATION & EXTENSION**

Use index laws to simplify algebraic expressions

Treat index laws as formulae (state which rule is being at each stage in a calculation)

Explain the difference between rational and irrational numbers as an introduction to surds

Prove that  $\sqrt{2}$  is irrational

Revise the difference of two squares to show why we use, for example  $(\sqrt{3}-2)$  as the multiplier to rationalise  $(\sqrt{3}+2)$

Link to work on circle measures (involving  $\pi$ ) and Pythagoras calculations in exact form.

**NOTES**

Link simplifying surds to collecting together like terms. eg  $3x + 2x = 5x$ , so therefore  $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Stress it is better to write answers in exact form, eg  $\frac{1}{3}$  is better than 0.333333.....

This is a non-calculator Unit, but students can check their answers on their calculators, most of which nowadays work in exact form anyway

A-Level C1 textbooks are a good source of extension questions on surd manipulation, some of which are algebraic

Useful generalisation to learn  $\sqrt{x} \times \sqrt{x} = x$

**Module** 28

**Time: 3 –5 hours**

**GCSE Tier: Higher**

**Contents: Circle theorems**

GM i Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

GM j Understand and construct geometrical proofs using circle theorems

**PRIOR KNOWLEDGE:**

Recall the words centre, radius, diameter and circumference

Have practical experience of drawing circles with compasses

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall the definition of a circle and identify (name) and draw the parts of a circle (12.4)
- Understand related terms of a circle (12.4)
- Draw a circle given the radius or diameter (12.4)
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point (31.1, 31.2)
- Understand and use the fact that tangents from an external point are equal in length (31.1, 31.2)
- Find missing angles on diagrams (31.1, 31.2)
- Give reasons for angle calculations involving the use of tangent theorems (31.1, 31.2)
- Prove and use the facts that: (31.3, 31.4)
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference
  - the angle in a semicircle is a right angle
  - angles in the same segment are equal
  - opposite angles of a cyclic quadrilateral sum to  $180^\circ$
  - alternate segment theorem
  - the perpendicular from the centre of a circle to a chord bisect the chord

**DIFFERENTIATION & EXTENSION**

Harder problems involving multi-stage angle calculations

**NOTES**

Any proof required will be in relation to a diagram, not purely by reference to a named theorem

Reasoning needs to be carefully constructed as ‘Quality of Written Communication’ marks are likely to be allocated to proofs



Module 30

Time: 4 – 6 hours

GCSE Tier: Higher

Contents: Vectors

GM cc Use vectors to solve problems

**PRIOR KNOWLEDGE:**

Vectors to describe translations  
Geometry of triangles and quadrilaterals

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that  $2\mathbf{a}$  is parallel to  $\mathbf{a}$  and twice its length (33.1 - 33.3, 33.5)
- Understand that  $-\mathbf{a}$  is parallel to  $\mathbf{a}$  and in the opposite direction (33.1 - 33.3, 33.5)
- Use and interpret vectors as displacements in the plane (with an associated direction) (33.1)
- Use standard vector notation to combine vectors by addition,  $\rightarrow \rightarrow \rightarrow$   
eg  $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$  and  $\mathbf{a} + \mathbf{b} = \mathbf{c}$  (33.4)
- Represent vectors, and combinations of vectors, in the plane (33.1 - 33.6)
- Solve geometrical problems in 2-D, eg show that joining the midpoints of the sides of any quadrilateral forms a parallelogram (33.6)

**DIFFERENTIATION & EXTENSION**

Harder geometric proof, eg Show that the medians of a triangle intersect at a single point  
Illustrate use of vectors by showing ‘Crossing the flowing River’ example or navigation examples.  
Vector problems in 3-D (for the most able)  
Use  $\mathbf{i}$  and  $\mathbf{j}$  (and  $\mathbf{k}$ ) notation

**NOTES**

Students often find the pictorial representation of vectors more difficult than the manipulation of column vectors.  
Geometry of a hexagon provides a rich source of parallel, reverse and multiples of vectors.  
Stress that parallel vectors are equal  
Link with like terms and brackets when simplifying  
Show there is more than one route round a geometric shape, but the answer simplifies to the same vector.  
Remind students to underline vectors or they will be regarded as just lengths with no direction  
Lengths are sometimes given in the ratio of 1:3; explain this concept carefully  
Some extension questions can be found in Mechanics 1 textbooks

Module 31

Time: 4 – 6 hours

GCSE Tier: Higher

Contents: Further graphs and functions

- A p Draw, sketch, recognise graphs of simple cubic functions, the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$ ,  
the function  $y = k^x$  for integer values of  $x$  and simple positive values of  $k$ ,  
the trigonometric functions  $y = \sin x$  and  $y = \cos x$
- A q Construct the graphs of simple loci

#### PRIOR KNOWLEDGE:

Linear functions 1  
Quadratic functions

#### OBJECTIVES

By the end of the module the student should be able to:

- Plot and recognise cubic, reciprocal, exponential and circular functions functions  $y = \sin x$  and  $y = \cos x$ , within the range  $-360^\circ$  to  $+360^\circ$  (see above) (21.2 - 21.4, 22.10)
- Use the graphs of these functions to find approximate solutions to equations, eg given  $x$  find  $y$  (and vice versa) (21.2 - 21.4, 22.10, 29.3)
- Find the values of  $p$  and  $q$  in the function  $y = pq^x$  given the graph of  $y = pq^x$  (21.2 - 21.4)
- Match equations with their graphs
- Recognise the characteristic shapes of all these functions (21.2 - 21.4, 22.10, 29.3)
- Construct the graphs of simple loci including the circle  $x^2 + y^2 = r^2$  for a circle of radius  $r$  centered at the origin of the coordinate plane (22.10)
- Find the intersection points of a given straight line with this circle graphically (22.10)
- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines (10.3, 22.10, 15.5)
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, or where the second equation is of the form  $x^2 + y^2 = r^2$  (22.10)

#### DIFFERENTIATION & EXTENSION

Explore the function  $y = e^x$  (perhaps relate this to  $y = \ln x$ )

Explore the function  $y = \tan x$

Find solutions to equations of the circular functions  $y = \sin x$  and  $y = \cos x$  over more than one cycle (and generalise)

This work should be enhanced by drawing graphs on graphical calculators and appropriate software

Complete the square for quadratic functions and relate this to transformations of the curve  $y = x^2$

#### NOTES

Make sure the students understand the notation  $y = f(x)$ , start by comparing  $y = x^2$  with  $y = x^2 + 2$  before mentioning  $y = f(x) + 2$  etc

Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit

Link with trigonometry and curved graphs

Module 32

Time: 3 – 5 hours

GCSE Tier: Higher

Contents: Transformations of functions

A v Transformation of functions

**PRIOR KNOWLEDGE:**

Transformations

**OBJECTIVES**

By the end of the module the student should be able to:

- Apply to the graph of  $y = f(x)$  the transformations  $y = f(x) + a$ ,  $y = f(ax)$ ,  $y = f(x + a)$ ,  $y = af(x)$  for linear, quadratic, sine and cosine functions (30.1 - 30.4)
- Select and apply the transformations of reflection, rotation, enlargement and translation of functions expressed algebraically (30.1 - 30.4)
- Interpret and analyse transformations of functions and write the functions algebraically (30.1 - 30.4)

**DIFFERENTIATION & EXTENSION**

Complete the square of quadratic functions and relate this to transformations of the curve  $y = x^2$

Use a graphical calculator/software to investigate transformations

Investigate curves which are unaffected by particular transformations

Investigations of the simple relationships such as  $\sin(180 - x) = \sin x$ , and  $\sin(90 - x) = \cos x$

**NOTES**

Make sure the students understand the notation  $y = f(x)$ , start by comparing  $y = x^2$  with  $y = x^2 + 2$  before mentioning  $y = f(x) + 2$  etc

Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit

Link with trigonometry and curved graphs



# Higher course objectives (1MA0)

## Number

- N a Add, subtract, multiply and divide whole numbers integers and decimals
- N a Multiply and divide fractions
- N b Order integers and decimals
- N b Order rational numbers
- N c Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Least Common Multiple, prime number and prime factor decomposition
- N d Use the terms square, positive and negative square root, cube and cube root
- N e Use index notation for squares, cubes and powers of 10
- N e Use index notation for squares, cubes and powers of 10
- N f Use index laws for multiplication and division of integer powers
- N f Use index laws for multiplication and division of integer, **fractional and negative** powers
- N g **Interpret, order and calculate with numbers written in standard index form**
- N h Understand equivalent fractions, simplifying a fraction by cancelling all common factors
- N i Add and subtract fractions
- N j Use decimal notation and recognise that each terminating decimal is a fraction
- N j Use decimal notation and recognise that each terminating decimal is a fraction
- N k Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
- N l Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
- N m Use percentage, **repeated proportional change**
- N n Understand and use direct **and indirect proportion**
- N o Interpret fractions, decimals and percentages as operators
- N p Use ratio notation, including reduction to its simplest form and its various links to fraction notation
- N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
- N r Use  $\pi$  in an exact calculation
- N r Calculations with surds
- N r **Use surds in exact calculations**
- N s Calculate upper and lower bounds
- N s Calculate upper and lower bounds
- N t Divide a quantity in a given ratio
- N u Approximate to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures
- N v Use a calculator efficiently and effectively

# Algebra

- A a Distinguish the different roles played by letter symbols in algebra, using the correct notation
- A b Distinguish in meaning between the words 'equation', 'formula', '**identity**' and 'expression'
- A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, **multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions**
- A d Set up and solve simple equations
- A d Set up and solve simple equations **including simultaneous equations in two unknowns**
- A e **Solve quadratic equations**
- A f Derive a formula, substitute numbers into a formula and change the subject of a formula
- A g Solve linear inequalities in one variable, and represent the solution set on a number line
- A g Solve linear inequalities in **two** variables, and represent the solution set on a **coordinate grid**
- A h Using systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
- A j Use linear expressions to describe the  $n^{\text{th}}$  term of an arithmetic sequence
- A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
- A l Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
- A m **Understand that the form  $y = mx + c$  represents a straight line and that  $m$  is the gradient of the line and  $c$  is the value of the  $y$ - intercept**
- A n **Understand the gradients of parallel lines**
- A o **Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions**
- A p **Draw, sketch, recognise graphs of simple cubic functions, the reciprocal function  $y = 1/x$  with  $x \neq 0$ , the function  $y = kx^n$  for integer values of  $x$  and simple positive values of  $k$ , the trigonometric functions  $y = \sin x$  and  $y = \cos x$**
- A q **Construct the graphs of simple loci**
- A r Construct linear functions from real-life problems and plot their corresponding graphs
- A r Construct linear, **quadratic and other** functions from real-life problems and plot their corresponding graphs
- A r Construct distance time graphs
- A s Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
- A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
- A u **Direct and indirect proportion (algebraic)**
- A v **Transformation of functions**

## Geometry

- GM a Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
- GM b Understand and use the angle properties of parallel lines, triangles and quadrilaterals
- GM c Calculate and use the sums of the interior and exterior angles of Polygons
- GM d Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
- GM e Recognise reflection and rotation symmetry of 2-D shapes
- GM f Understand congruence and similarity
- GM g Use Pythagoras' theorem in 2-D and 3-D
- GM h **Use the trigonometric ratios and the sine and cosine rules to solve 2-D and 3-D problems**
- GM h Use the sine and cosine rules to solve 2-D and 3-D problems**
- GM i Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- GM j **Understand and construct geometrical proofs using circle theorems**
- GM k Use 2-D representations of 3-D shapes
- GM l Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, **fractional or negative** scale factor and distinguish properties that are preserved under particular transformations
- GM m Use and interpret maps and scale drawings
- GM n Understand **and use** the effect of enlargement for perimeter, area and volume of shapes and solids
- GM o Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
- GM p Convert measurements from one unit to another
- GM p Convert measurements from one unit to another
- GM p Convert between volume measures, including cubic centimetres and cubic metres
- GM q Make sensible estimates of a range of measures
- GM s Understand and use compound measures
- GM v Use straight edge and a pair of compasses to carry out constructions
- GM w Construct loci
- GM x Calculate perimeters and areas of shapes made from triangles and rectangles **or other shapes**
- GM x Calculate perimeters and areas of shapes made from triangles and rectangles **or other shapes**
- GM y Calculating the area of a triangle using  $\frac{1}{2} ab \sin C$**
- GM z Find circumferences and areas of circles
- GM z Find surface area of a cylinder
- GM aa Calculate volumes of right prisms and shapes made from cubes and cuboids
- GM bb **Solve mensuration problems involving more complex shapes and solids**
- GM cc **Use vectors to solve problems**

## Statistics and Probability

- SP a Understand and use statistical problem solving process/handling data cycle
- SP b Identify possible sources of bias
- SP c Design an experiment or survey
- SP d Design data-collection sheets distinguishing between different types of data
- SP e Extract data from printed tables and lists
- SP f Design and use two-way tables for discrete and grouped data
- 
- SP g Produce charts and diagrams for various data types
- SP g Produce charts and diagrams for various data types
- SP h Calculate median, mean, range, **quartiles and interquartile range**, mode and modal class
- SP i Interpret a wide range of graphs and diagrams and draw conclusions
- SP i Interpret a wide range of graphs and diagrams and draw conclusions
- SP j Present findings from databases, tables and charts
- SP k Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
- SP l Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts
- SP l Compare distributions and make inferences
- SP m Understand and use the vocabulary of probability and probability scale
- SP n Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
- SP o List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
- 
- SP p Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- SP q **Know when to add or multiply two probabilities: when  $A$  and  $B$  are mutually exclusive, then the probability of  $A$  or  $B$  occurring is  $P(A) + P(B)$ , whereas when  $A$  and  $B$  are independent events, the probability of  $A$  and  $B$  occurring is  $P(A) \times P(B)$**
- SP r **Use tree diagrams to represent outcomes of compound events, recognising when events are independent**
- SP s Compare experimental data and theoretical probabilities
- SP t Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
- SP u Use calculators efficiently and effectively, including statistical functions

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